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DYNAMICS OF STEADY OCEAN CURRENTS IN THE LIGHT OF EXPERIMENTAL FLUID MECHANICS

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INTRODUCTION

The present investigation may be regarded as a part of a systematic effort to introduce into meteorology and physical oceanography methods and results which for a number of years have contributed to the rapid growth and increasing practical significance of experimental fluid mechanics. This science has recognized that the exact character of the forces controlling the motion of a turbulent fluid is not known and that consequently there is very little justification for a purely theoretical attack on problems of a practical character. For this reason fluid mechanics has been forced to develop a research technique all of its own, in which the theory is developed on the basis of experiments and then used to predict the behavior of fluids in cases which are not accessible to experimentation.

In oceanography it has long been regarded as an axiom that the movements of the water are controlled by three forces, the horizontal pressure gradient, the deflecting force, and the frictional force resulting from the relative motion of superimposed strata. It is significant that thirty-five years of intensive theoretical work on this basis have failed to produce a theory capable of explaining the major features of the observed oceanic circulation below the pure drift current layer.

The present investigation considers a force which has been completely disregarded by theoretical investigators although its existence has been admitted implicitly by practically everyone who has approached physical oceanography from the descriptive side, namely the frictional force resulting from large-scale horizontal mixing. The introduction of this force permits us to see how motion generated in the surface layers may be diffused and finally dissipated without recourse to doubtful frictional forces at the bottom of the ocean.

A great number of practical hydrodynamic investigations of the observed oceanic current systems consist mainly in velocity calculations with the aid of the circulation theorem. Without denying the great practical value of the circulation theorem, the present investigation endeavors to emphasize a fact which by this time should have been generally accepted but which it not always kept in mind, namely the impossibility of drawing any conclusions regarding the cause of oceanic motions from the ordinary routine application of the circulation theorem.

In the first part of the paper the principal imperfections of the present theory for the oceanic circulation are set forth. Frictional forces due to horizontal mixing are then introduced and the effect of the earth's rotation on the horizontal eddy velocities analyzed. Tollmien's theory for the mixing along the edges of a steady stream moving through a resting fluid is then discussed and certain experimental verifications are described. With the aid of a principle first stated by G. I. Taylor, Tollmien's results are applied to current systems subject to a deflecting force. Finally certain important modifications resulting from the stratification in the ocean are treated.

In the second part of the paper an attempt is made to trace the mixing between the Gulf Stream and its surroundings with the aid of the observed distribution of temperature, salinity and oxygen. The results of this qualitative analysis seem to bear out the theoretical predictions.

The theory set forth is utterly incomplete, and serious objections may be raised against the looseness of the reasoning on which it is based. Nevertheless, the author believes that it may serve as a useful working hypothesis, since its predictions refer to an idealized stratified ocean and not to a non-existent homogeneous medium.

The present paper is to a large extent the result of fruitful cooperation between a number of persons. Dr. H. Peters of the Massachusetts Institute of Technology not only carried out certain experimental tests of Tollmien's theory but also, in a number of discussions, directed my attention to various investigations bearing on the relative merits of the momentum transfer and the vorticity transfer theories.

Mr. C. O'D. Iselin of the Woods Hole Oceanographic Institution has contributed his vast knowledge of the hydrographic conditions in the North Atlantic. Without his active cooperation it would have been impossible to carry through the investigation to a point where it could be tested against observations. Several of the conclusions here derived from purely theoretical considerations have already been reached by Mr. Iselin from a study of the hydrographic data collected by the Woods Hole Oceanographic Institution.

Mr. H. R. Seiwell's investigations of the oxygen distribution in the North Atlantic have been particularly helpful and are responsible for the choice of oxygen as an indicator of horizontal mixing.

The author is indebted to Dr. H. B. Bigelow for various helpful suggestions.

A brief account of the principal theoretical results presented below was given before the annual meeting of the Institute of the Aeronautical Sciences in New York, January, 1936.

At the date of writing this introduction, Dr. A. E. Parr of Yale University informs me that he has been led to conclusions of substantially the same nature as some of the ones here presented, through a study of recent hydrographic data from the Caribbean. Dr. Parr's results will be published in *Journal du Conseil.*

Woods Hole, July 15, 1936

I. THEORETICAL DISCUSSION

A. Formulation of Problem

Anybody who has attempted to construct, for his own satisfaction, theoretical working models of the permanent current system in the ocean or in the air or of some of the apparently steady phenomena of the secondary circulation, sooner or later runs up against the apparent impossibility of finding forces capable of producing, in the interior of the media under consideration, horizontal convergence or divergence on a scale comparable to that which actually must occur in nature. In cyclonic regions, surface friction produces an easily observed transport of air across the isobars towards lower pressure. Since the gradient wind supposed to prevail at higher levels is very nearly free from divergence there is apparently no way in which the accumulating surface air may be removed. One would therefore expect a rapid decay or filling up of these low pressure systems. Nevertheless, particularly the occluded cyclones of higher latitudes and the hurricanes of lower latitudes often seem to be characterized by a condition of approximate dynamic equilibrium.

A similar problem appears in the interpretation of the horizontal circulation of the ocean. The permanent anticyclonic wind system of the North Atlantic Ocean produces a steady accumulation of surface water in lower latitudes and a corresponding slope of the sea surface. The resulting gradient current system should be very nearly free from horizontal divergence and thus incapable of re-establishing equilibrium. To avoid this difficulty Ekman,[1] in his general theory of the circulation of a homogeneous ocean, assumes that the bottom friction is so strong that it produces a divergence sufficient to offset the wind-produced surface convergence. It is easily shown that the bottom friction required for this purpose must be of the same order of magnitude as the surface friction.

Ekman's solution implies that bottom water and surface water are equivalent. In each region of surface convergence and bottom divergence there must be a descending motion, in each region of surface divergence and bottom convergence there must be an ascending motion, so that bottom and surface water continually replace each other. While this may be acceptable in the ideal case of a homogeneous ocean, it is in sharp disagreement with observed conditions in the real, stratified ocean.

According to Ekman's theory the equatorial side of the subtropical Highs must be characterized by such accumulation of surface water. The observed steep thermocline in these regions shows that the accumulation and sinking of surface water must cease within a depth of a few hundred meters, in contrast with the theoretical prediction, although there are definite indications that strong horizontal convergence and sinking must occur in these upper layers.

Since the vertical circulation does not extend all the way down it may be argued that the water, because of its stratification, has a cellular structure, each cell being separated through approximately horizontal surfaces of discontinuity from the cells above and below. Each boundary surface would then act as a "false" bottom and each cell would have a practically independent circulation. In order to have steady conditions and zero horizontal divergence in each cell, it would be necessary for the shearing stresses at each boundary to be of the same order of magnitude as those at the surface. This stress distribution would produce a much stronger circulation in the bottom cell than indicated by available data. Furthermore, for each new cell introduced the surface current velocity is raised, so that the suggested scheme most likely would produce impossible surface velocities.

It is no doubt possible to overcome some of these difficulties locally by considering the deviations from gradient flow associated with inertia forces. This is the line of attack followed by Ekman in his latest investigations. It is as yet impossible to estimate completely the extent to which this much needed extension of the theory will eliminate the difficulties listed above. However, in this connection the following comment is pertinent:

The horizontal circulation of the southern half of the North Atlantic may be represented as a gigantic *stationary* anticyclonic eddy maintained by the permanent anticyclonic wind system over the same area. Since the mean motion is steady, the mean total torque round a vertical axis must vanish. In Ekman's theory this is accomplished through the introduction of frictional forces at the bottom, the torque of which balances the wind torque. The consideration of inertia forces in no way removes the need for this balancing frictional force at the bottom. Actually observations indicate that the motion near the bottom is vanishingly small and thus incapable of producing frictional forces of any significance.

An inspection of a current chart for the North Atlantic indicates that strong eddying motion occurs at many places along the borders of the basin. *Thus it seems possible that the required balance may be established through frictional forces originating on the continental slopes and transmitted through the water as shearing stresses acting on vertical surfaces parallel to the horizontal current components.* For the sake of brevity shearing stresses of this type will here be referred to as lateral stresses, while the designation normal stresses is reserved for stresses acting on horizontal surfaces and produced by the vertical variation in horizontal velocity. It is evident, from a study of the relative horizontal and vertical dimensions of atmospheric and oceanic systems, that the lateral stresses must be many times larger than the normal ones if they are to be of any dynamic significance.

The idea that momentum may be transferred horizontally through turbulence is not new. In a much-discussed paper published in 1921 Defant[2] assumed that the travelling cyclones and anticyclones may be regarded as turbulent elements superimposed on the mean circulation of the atmosphere in middle latitudes. Defant used this conception of the general circulation to compute the advective transfer of heat from the equator to the poles. However, in Defant's case the eddying components are quite large compared to the mean motion, so large, in fact, that the mean motion of the air is often completely obscured by the presence of the eddying motion. It is doubtful that these large eddies derive their energy from the mean motion, and perhaps more likely that the reverse is true. Thus it appears desirable to select for study a steady fluid system characterized by a well-established primary mean motion and to determine the rôle played by lateral shearing stresses in the dynamics of this system.

Richardson and Proctor[3] have investigated horizontal diffusion in atmospheric currents by means of the scattering of volcanic ash and the scattering of small toy balloons. For distances ranging between 3 km. and 86 km. these authors obtained values of the horizontal diffusivity varying between $2 \cdot 10^6$ and $1.3 \cdot 10^9$ cm.^{2}/sec. It is reasonable to assume that the turbulent mechanism responsible for this scattering must produce an equally intensive lateral diffusion of horizontal momentum. The lateral stresses introduced above are simply a measure of this lateral eddy transport of momentum.

If Richardson's and Proctor's coefficients are expressed as eddy-viscosities, they range from $2.5 \cdot 10^3$ to $1.6 \cdot 10^6$ grams/cm.sec. Thus they are intermediate in magnitude between the values obtained from the study of vertical wind gradients, 10^2 grams/cm.sec., and the values obtained by Defant from an analysis of the general circulation as a turbulent process, 10^8 grams/cm.sec. In Defant's analysis of the general circulation the individual turbulent elements are supposed to consist of travelling cyclones and anticyclones or, more properly, of large bodies of air from different source regions. The diffusion process measured by Richardson and Proctor, and studied from another point of view in the present paper, deals with phenomena within a single air or ocean current and along its boundaries. It presupposes the existence of eddies whose dimensions must be measured in fractions of a kilometer up to, perhaps, twenty or thirty kilometers. The remarkable uniformity in air mass characteristics so often observed in our aerological data suggests that horizontal diffusion on such a large scale must occur with great regularity in the atmosphere. It is rather surprising then to find that the dynamic consequences of this horizontal diffusion mechanism never have been investigated.

Before proceeding, it may be worth while to point out how lateral shearing stresses affect the horizontal divergence. On the northern hemisphere, steady, non-accelerated motion in the atmosphere or in the ocean is characterized by the fact that to a given horizontal force P there corresponds a horizontal momentum M directed 90° to the right from P and having the value

$$M = \frac{P}{2\omega \sin L}, \tag{1}$$

where L is the latitude and ω is the angular velocity of the earth. As an illustration, consider a vertical air column in a field of straight, parallel isobars. This column is acted upon by the horizontal pressure gradient and by the frictional force between the air and the ground. It is evident that the component of its momentum across the isobars must correspond to the component of ground friction parallel to the isobars. If the same column of air is subject not only to normal stresses but also to suitable lateral shearing stresses, the resultant force along the isobar direction and thus also the total flow across the isobars may be made to vanish.

Because of the earth's rotation, the effect of the normal shearing stresses originating at a horizontal boundary vanish within a relatively short vertical distance. Outside these shallow boundary layers the velocities vary only slowly along the vertical, at least when there is steady motion and when the medium considered is in barotropic equilibrium; thus we are permitted to assume that the lateral shearing stresses are reasonably independent of the vertical coordinate through fairly deep strata. This effect of the earth's rotation simplifies a separation of the effects of lateral and normal stresses; such a separation, on the other hand, is not readily possible in the case of small-scale hydraulic experiments.

The balance of forces in a horizontal direction normal to the mean motion, which consists in an equilibrium between deflecting force and horizontal pressure gradient, is not materially affected by the presence of lateral stresses. *This balance, which for the atmosphere takes the form of the ordinary gradient wind relationship and which is also utilized for so called "dynamic velocity calculations" of ocean currents, does not prescribe a definite velocity profile across the current.* More specifically, if we consider the mass distribution in a certain vertical plane, it is always possible to find a distribution of velocities normal to this plane such that the resulting deflecting force everywhere balances the horizontal pressure gradient resulting from the mass distribution (distribution of solenoids). Conversely, it is always possible to find a mass distribution in a vertical plane such that the resulting horizontal pressure gradient balances the deflecting force associated with an arbitrary distribution of velocities normal to the plane.

On the other hand, the effect of lateral stresses acting in the direction of the motion must be to produce certain characteristic transversal velocity profiles. If, then, through an analysis of available observations, the existence of certain preferred atmospheric or oceanic current profiles is established, which profiles from a comparison with completely controlled laboratory experiments appear to be the result of frictional forces (lateral stresses), we are reasonably justified in assuming that the associated mass (solenoid) distribution in a transversal plane must be regarded as a result rather than as a cause of the motion. This point is stressed here since there seems to be a tendency on the part of many oceanographers to regard the mass distribution, which serves as a starting point in all dynamic calculations of so-called "convection" currents, as their cause. As a matter of fact, it is easy enough to show how, on a rotating globe, solenoids may be generated through mechanical means.[4] It is possible to develop criteria for the separation of such secondary *dynamic* solenoids from the *thermal* solenoids, which are the ultimate cause of all motion in the atmosphere. Thus one should expect to find the vertical correlation curve between temperature and salinity to be independent of location in an ocean current section whose solenoids are dynamic in origin. Similarly, in a section across a steady air current in which the solenoids are of secondary character, the vertical correlation between specific humidity and potential temperature ought to be reasonably constant. Illustrative examples will be furnished in the second part of this investigation.

B. Effect of the Earth's Rotation on Lateral Stresses

The evaluation of lateral stresses in the air or in the ocean brings up another problem of general significance, namely, the effect of curvature and of the earth's rotation on the turbulent exchange of momentum between fluid strata moving side by side. It thus forces us to choose between the "vorticity-transport" theory developed by Taylor[5] and the "momentum-transport" theory developed by Prandtl.[6] Taylor has pointed out that the structure of straight fluid current systems may be interpreted equally well with the aid of the one as with the aid of the other of these two theories but that, in the case of curved flow or flow in rotating systems the two theories lead to mutually exclusive results. It seems appropriate to follow up this comment of Taylor's with an analysis of the predictions of the two theories in as far as atmospheric and oceanic motion is concerned.

As a starting point we choose a steady terrestrial fluid system rotating cyclonically relative to the surface of the earth around a certain vertical axis A. The rotation of the earth itself may be resolved into a rotation around A and a rotation around an axis normal thereto. The latter rotation is without significance in the present connection. The relative linear velocity at a distance r from the axis is given by v. It we designate by

$f = 2\omega \sin L$ the Coriolis parameter, it follows that the absolute linear velocity V around the axis has the value

(2) $$V = v + \tfrac{1}{2} fr.$$

The absolute angular momentum around A is given by

(3) $$\Omega = rV = rv + \tfrac{1}{2} fr^2.$$

According to the momentum transfer theory each element displaced along the radius tends to retain its original angular momentum. Thus an element displaced from r to $r+l$ will produce, in its new position, a deviation of the observed angular momentum from the mean, given by

(4) $$\Omega' = \Omega_r - \Omega_{r+l} = -l \frac{\partial \Omega}{\partial r},$$

and consequently a deviation of the tangential velocity from the mean, given by

(5) $$v' = -\frac{l}{r} \frac{\partial \Omega}{\partial r}.$$

Assuming equipartition of eddy energy it follows that the shearing stress is given by

(6) $$\tau = -\rho\, \overline{u'v'} = \rho\, \overline{u'l} \frac{1}{r} \frac{\partial \Omega}{\partial r} = \rho \frac{l^2}{r^2} \left(\frac{\partial \Omega}{\partial r} \right)^2.$$

In this expression u' represents the radial (eddying) velocity. Thus the momentum transfer theory indicates that the shearing stress vanishes when the absolute angular momentum is independent of the distance from the axis. One may now introduce the relative motion in the above expression. The result is

(7) $$\tau = \rho l^2 \left(\frac{\partial V}{\partial r} + \frac{V}{r} \right)^2 = \rho l^2 \left(\frac{\partial v}{\partial r} + \frac{v}{r} + f \right)^2.$$

If the radius of curvature is sufficiently large, the above expression reduces to the form

(8) $$\tau = \rho l^2 \left(\frac{\partial v}{\partial x} + f \right)^2,$$

where x is a horizontal coordinate counted positive in a direction 90° to the right of the direction in which the current is flowing. Thus the momentum transfer theory indicates that in a straight air or ocean current the lateral shearing stresses vanish when the velocity decreases towards the right edge of the current at the rate

(9) $$\frac{\partial v}{\partial x} = -f.$$

This is a very steep rate, which in middle latitudes (43°) corresponds to a rate of shear of 1 cm.p.s. in 100 meters. Such horizontal rates of shear are hardly ever observed in the ocean and in the atmosphere they occur only along fronts. Thus, according to the momentum transfer theory, the right edge of a current always tends to accelerate the left edge, even though the velocity to the right may be considerably less than the velocity to the left. In particular, a broad, uniform current would be subject to shearing stresses tending to produce a velocity profile with a steep drop in velocity towards the right of the current.

This result of the momentum transfer theory may be obtained in a different way, which brings out another discrepancy between the two theories. Consider a straight current flowing in the direction of the y-axis. There is equilibrium between the horizontal pressure gradient and the deflecting force corresponding to the mean velocity $\bar{v}$. In the course of the turbulent motion, individual elements will move across the stream. If the horizontal velocity components of the moving elements are designated by u and v, their equations of motion will be

(10) $$\frac{du}{dt}=fv-\frac{1}{\rho}\frac{\partial p}{\partial x}=f(v-\bar{v})$$

(11) $$\frac{dv}{dt}=-fu.$$

The second of these two equations may be integrated at once and gives

(12) $$v-v_0=-f(x-x_0)=-fl,$$

where l means the displacement of the element cross-stream and the subscript o refers to the initial state. We may assume that the element originally had the same velocity downstream as its surroundings so that

(13) $$v_0=\bar{v}_0.$$

Thus, as a result of the displacement l, the element will appear in its new position with a velocity in excess of that of the surroundings. This excess is given by

(14) $$v'=v-\bar{v}_l=\bar{v}_0-fl-\bar{v}_l=-l\left(f+\frac{\partial \bar{v}}{\partial x}\right).$$

The lateral shearing stress may be computed from

(15) $$\tau=-\rho\overline{u'v'}=\rho\overline{lu'}\left(f+\frac{\partial \bar{v}}{\partial x}\right).$$

Again it appears that the shearing stress disappears, not when the current is uniform, but when

(16) $$\frac{\partial \bar{v}}{\partial x}=-f.$$

Furthermore, the rotation of the earth would appear to produce strong stabilizing forces tending to suppress turbulence. If we insert the expression

(17) $$v=\bar{v}_0-fl$$

in the first equation of motion (10), we find

(18) $$\frac{du}{dt}=f(\bar{v}_0-fl-\bar{v}_l)=-fl\left(f+\frac{\partial \bar{v}}{\partial x}\right).$$

Since the last factor is positive for practically all atmospheric or oceanic systems it follows that the acceleration du/dt is negative. Thus elements moving cross-stream are subject to a strong restoring force. Per unit mass and displacement this force has the value

$$RF = f\left(f + \frac{\partial \bar{v}}{\partial x}\right). \tag{19}$$

Richardson[7] has introduced a non-dimensional parameter (P) which may serve as a measure for the effectiveness of stabilizing forces in suppressing turbulence. It is obtained by dividing the stablizing force through the square of the vorticity. The significance of this quantity is that it measures the ratio between the amount of eddy energy lost through work against stabilizing forces and the amount of eddy energy produced through the work of the eddy shearing stresses. On the basis of the momentum transfer theory this ratio has the value

$$P = \frac{f\left(f + \frac{\partial \bar{v}}{\partial x}\right)}{\left(f + \frac{\partial \bar{v}}{\partial x}\right)^2} = \frac{f}{f + \frac{\partial \bar{v}}{\partial x}} \approx 1, \tag{20}$$

which is sufficiently high to suppress lateral turbulence in a very efficient way.

The momentum transfer theory, as applied to a symmetric rotating system, assumes that individual elements retain their original absolute angular momentum during radial displacements. Taylor (l.c.) has pointed out that this assumption implies that local pressure gradients resulting from the displacements can be neglected. This may not always be true. On the other hand, we do know that the elements in the absence of viscosity retain their original vorticity regardless of displacements. The expression for the frictional force should be such as to take cognizance of this fact. The effect of the eddies is to produce a transport of vorticity from regions of high to regions of low vorticity. Since the gradient of vorticity is not changed by a rotation of the system as a whole around a fixed axis, this rotation simply having the effect of adding a constant amount of vorticity to each point in the system, it appears that the shearing stresses must have such a form that the addition or subtraction of a solid rotation does not change their value. In the case of a radially-symmetric, terrestrial fluid system, rotating around a vertical axis, this is the case if we assume

$$\tau = \rho l^2 \left(\frac{\partial V}{\partial r} - \frac{V}{r}\right)^2, \tag{21}$$

where V is the absolute velocity. With the aid of (2) we may introduce the velocity v relative to the surface of the earth and obtain

$$\tau = \rho l^2 \left(\frac{\partial v}{\partial r} - \frac{v}{r}\right)^2. \tag{22}$$

For straight currents this expression reduces to

$$\tau = \rho l^2 \left(\frac{\partial v}{\partial x}\right)^2, \tag{23}$$

where x again is a horizontal coordinate pointing cross-stream to the right of the direction of flow. From this expression the reference to the rotation of the earth has disappeared.

Without attempting to attack the general question of the effect of local pressure gradients, it may be stated that the present problem offers a particularly good opportunity for its study. Let us assume that the lateral stresses in the ocean are caused by vertical fluid columns moving cross-stream in an irregular fashion. To study the effect of local pressure gradients, assume furthermore that each column has a circular cross-section with a radius a and is so deep that the motion of the surrounding displaced water is reasonably free from horizontal convergence or divergence. Under these assumptions the motion of the fluid outside the cylinder will be of the type known from potential theory, but the pressure distribution will differ from the one given by the classical solution. If the fluid at some distance from the cylinder is at rest or in uniform horizontal motion (relative to the earth) it is found that the moving cylinder is acted upon by two forces. The first is the deflecting force, which, per unit length of the cylinder, has the absolute value

$$DF = \rho' \pi a^2 f u. \tag{24}$$

In this expression ρ' is the density of the fluid in the cylinder and u is its velocity. This deflecting force is horizontal and directed 90° to the right from the velocity u. The second force results from the pressure distribution in the surrounding fluid and has the value

$$PF = \rho \pi a^2 f u, \tag{25}$$

where ρ is the density of the displaced fluid. This second force acts in the opposite direction to the first. If the two densities are equal, the two forces balance each other.

The creation of horizontal pressure gradients around the moving cylinder requires changes in level at the free surface and thus also horizontal divergence, but the amount of this divergence can be made negligibly small by making the cylinders sufficiently deep. Thus the motion of the cylinder is not affected by the rotation of the earth, contrary to the assumption underlying the momentum transfer theory.

C. "Coriolian" Pressure Gradients

Returning to the main topic we may say that in the absence of horizontal convergence and divergence, a moving fluid portion will be subjected to horizontal pressure gradients which will completely offset the deflecting force. This result was first obtained by Taylor (l. c., p. 696) in 1932 and expressed by him in the following fashion: "If ψ is the stream function at any instant of any two-dimensional motion of a viscous incompressible fluid, then the whole system may be rotated with uniform angular velocity Ω about an axis perpendicular to the plane of motion, and a motion relative to the rotating axes identical in every respect with the original motion is possible. If p is the pressure corresponding with the original motion, the pressure when the whole system is rotated is $p+2\rho\,\Omega\psi+\frac{1}{2}\rho\,\Omega^2 r^2$, where r is the distance from the centre of rotation. The stresses due to viscosity are unaltered by the rotation as also are the stresses due to turbulence."

In our case the term $\frac{1}{2}\rho\,\Omega^2 r^2$, obtained from the centrifugal force associated with the rotation of the coordinate system, drops out since the centrifugal force is offset by a component of the true acceleration of gravity. The quantity $2\,\Omega$ occurring in the above quotation from Taylor is identical with the Coriolis parameter f in our notation. Thus sufficiently deep currents may be analyzed as if the earth were not rotating, provided we subtract from the acting forces the secondary "Coriolian" pressure gradients which represent the reaction of the fluid to the rotation of the earth. These gradients are given by

$$(26) \qquad -\frac{\partial p_c}{\partial x} = -\rho f v$$

$$(27) \qquad -\frac{\partial p_c}{\partial y} = +\rho f u.$$

It is apparent that a pressure field satisfying these equations always can be found provided

$$(28) \qquad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$

that is, provided the motion is free from horizontal convergence and divergence. The Coriolian pressure gradients are normally many times larger than the dynamically more significant residuals. This fact enables us to compute currents with a reasonable degree of accuracy from the total pressure gradients.

The preceding discussion should serve to emphasize the complex, and to a very large extent secondary, character of the horizontal pressure distribution. With respect to the oceanic circulation it seems particularly appropriate to emphasize the following point: The swift currents in the ocean troposphere owe their existence, directly or indirectly, to wind friction. In the case of such motions, momentum may be transferred from layer to layer through shearing stresses, and the pressure distribution may then be entirely secondary in character. A simple illustration is furnished by a fluid contained between two concentric vertical cylinders and having one free surface. If the outer cylinder is set in motion it will gradually transmit its momentum to deeper and deeper fluid strata through shearing stresses. A radial pressure gradient (sloping free surface) gradually develops as a reaction to the centrifugal force but plays no rôle in the transfer of momentum.

Thus, in the case of terrestrial systems which are free from convergence or divergence, it is possible to eliminate the influence of the rotation of the earth through the balance between deflecting force and the Coriolian pressure field. The remaining terms, which have received relatively small attention in theoretical meteorological or oceanographical literature, are the ones that really give some information concerning the dynamics of the system.

D. Wake Stream Theory

We shall now consider the balance of forces in a steady, deep current flowing through an ocean basin of uniform depth. The axis of the current coincides with the x-axis. It is assumed that the motion is two-dimensional, horizontal and, because of the depth of the basin, very nearly free from horizontal divergence. Under these conditions and omitting insignificant terms, the equations for the relative motion are

(29) $$\rho\frac{du}{dt}=\rho f v-\frac{\partial p}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}$$

(30) $$\rho\frac{dv}{dt}=-\rho f u-\frac{\partial p}{\partial y}+\frac{\partial \tau_{yx}}{\partial x}.$$

τ_{xy} represents the x-component of the shearing stress acting across a plane normal to the y-axis. τ_{yx} may be interpreted in a similar fashion.

To eliminate the rotation, subtract the deflecting forces and the balancing Coriolian pressure gradients (26), (27) from the forces acting on the system. The result is

(31) $$\rho\frac{du}{dt}=-\frac{\partial p_r}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}$$

(32) $$\rho\frac{dv}{dt}=-\frac{\partial p_r}{\partial y}+\frac{\partial \tau_{yx}}{\partial x},$$

in which equation the "residual" pressure p_r is given by

(33) $$p_r=p-p_c.$$

With the aid of these equations the motion may be analyzed as if the basin were at rest, and the total pressure were given by p_r.

Consider now a current moving under its own momentum and produced by discharging water into the basin through a jet. The theory for such a current system was first developed by Tollmien.[8] The two-dimensional case has been studied experimentally by Förthmann[9] and recently, at the author's suggestion, by Peters and Bicknell.[10] Tollmien's theory for the symmetrical two-dimensional "wake stream" will be outlined below.

With the aid of the equation of continuity the first equation of motion (31) may be transformed and gives

(34) $$\rho\frac{\partial u^2}{\partial x}+\rho\frac{\partial uv}{\partial y}=-\frac{\partial p_r}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}.$$

Assuming that the current has definite boundaries, defined by the condition that u and τ_{xy} both vanish, we may integrate this equation with respect to y and obtain

35) $$\frac{\partial}{\partial x}\int \rho u^2 dy=-\int\frac{\partial p_r}{\partial x}dy.$$

The integration extends across the entire width of the current. Theory and observations show that the term on the right side of this equation is small. Thus, as a first approximation

(36) $$\int \rho u^2 dy=\text{constant},$$

i.e. *the momentum transport through any transversal section is approximately constant.* The experimental verification of this statement will be furnished below.

The approximate constancy of the momentum transport implies that

(37) $$\frac{\partial p_r}{\partial x}=0.$$

Consequently the equation of motion reduces to the form

(38) $$\rho \frac{\partial u^2}{\partial x} + \rho \frac{\partial uv}{\partial y} = \frac{\partial \tau_{xy}}{\partial y}.$$

In order to integrate this equation, Tollmien assumes that the shearing stress is given by

(39) $$|\tau_{xy}| = \rho l^2 \left(\frac{\partial u}{\partial y}\right)^2,$$

and the mixing length l by

(40) $$l = cx,$$

where c is a constant. One solution is then obtained by assuming that the stream function ψ has the form

(41) $$\psi = \sqrt{x}\, F(\eta) \equiv \sqrt{x}\, F\left(\frac{y}{x}\right),$$

where y is counted from the axis of the symmetric wake stream and increases to the left. Thus

(42a) $$u = \frac{\partial \psi}{\partial y} = \frac{1}{\sqrt{x}} F'$$

(42b) $$v = -\frac{\partial \psi}{\partial x} = -\frac{1}{\sqrt{x}} [\tfrac{1}{2} F - \eta F'].$$

$$\left(F' = \frac{dF}{d\eta}\right)$$

Since the boundaries of the wake stream are defined by the condition that u and τ_{xy}, i.e. $\partial u/\partial y$, vanish, it follows that these boundaries must coincide with the straight lines

(43) $$\eta = \eta_1, \ \eta = \eta_2 = -\eta_1,$$

where y_1 and y_2 are simultaneous roots to the equations

(44) $$F' = 0, \ F'' = 0.$$

It is easily seen that the above expression for u makes the momentum transport constant. If we integrate the equation of motion with respect to y, it follows that

(45) $$\rho \int_{y_1}^{y} \frac{\partial u^2}{\partial x}\, dy + \rho uv = \tau_{xy},$$

where y_1 refers to the right boundary of the wake stream. Substitution gives

(46) $$FF' = 2c^2 F''^2.$$

A first integral to this equation is obtained without difficulty, but the final solution is best expressed through development in series. One of the two integration constants is determined from the fact that v, and consequently F, must vanish in the axis of the current ($\eta = 0$). The second constant is needed to give the momentum transport its prescribed value and enters as a factor with which the expression for F is multiplied.

The mass transport T through a given section is given by

(47) $$T=\rho\int u dy=\rho\sqrt{x}\,[F(\eta_2)-F(\eta_1)].$$

Thus the mass transport increases downstream while the momentum transport remains constant. This evidently implies that there must be an inflow towards the wake stream from the surrounding fluid. The magnitude of the inflow may be determined by computing the values of v for η_2 and η_1, the boundaries of the current. The width of the current, b, is obtained from

(48) $$b=y_2-y_1=x(\eta_2-\eta_1).$$

Thus the current width increases downstream.

It appears from the expression for the mass transport (47) that this quantity vanishes for $x=0$. Since all wake stream experiments are made with jets of finite dimensions and finite outflow it follows that the origin for the x-coordinate must be placed a short distance inside the jet.

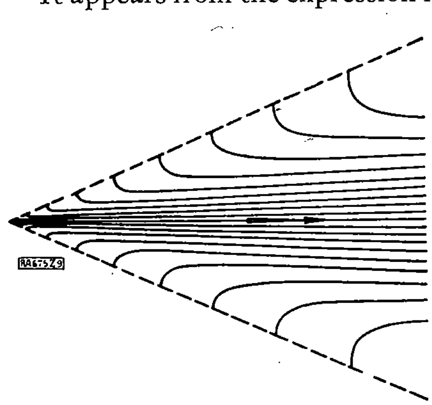

FIG. 1.—Theoretical stream lines in two-dimensional wake stream, according to Tollmien.

Fig. 1 shows the stream lines according to Tollmien and Fig. 2 and Fig. 3 give the theoretical distribution of transversal and axial velocities. Fig. 4 gives a non-dimensional representation of some of the results of Peters' and Bicknell's measurements. Finally in Fig. 5 their observed maximum velocities have been plotted against x. From the two last diagrams it may be inferred that in these measurements the momentum transport was very nearly constant.

The angular spread of the wake stream seems to vary greatly. It depends apparently upon the character of the jet and of the flow as it leaves the jet, but also on conditions in the surrounding fluid. In Peters' and Bicknell's case it varied between 8° and 14°. The angular spread varies in the same sense as the constant c introduced above. There are some indications that c decreases with increasing Reynolds number.

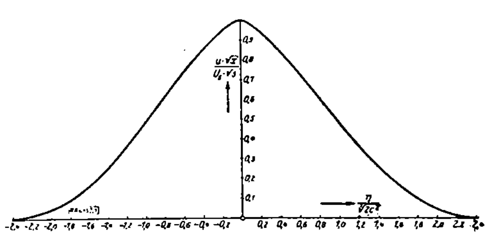

FIG. 2.—Non-dimensional representation of theoretical distribution of axial velocities in two-dimensional wake stream, according to Tollmien.

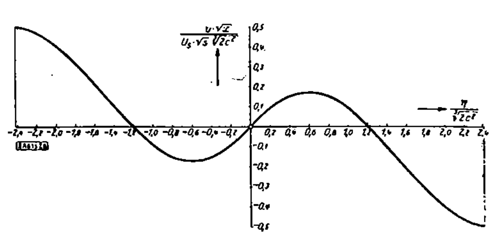

FIG. 3.—Non-dimensional representation of the theoretical distribution of transversal velocities in two-dimensional wake stream, according to Tollmien.

It is of interest to determine the distribution of vorticity within the wake stream. It is given by

(49)
$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y},$$

or, after substitution of the expressions for u and v,

(50)
$$\zeta = \frac{1}{x\sqrt{x}}\left[\tfrac{1}{4}F - \eta F' - (1+\eta^2)F''\right].$$

Along the boundaries of the wake stream u and $\partial u/\partial y$ vanish. Consequently F' and F'' vanish. Since $v \neq 0$ along the boundaries it follows (from 42b) that $F \neq 0$ along the edges

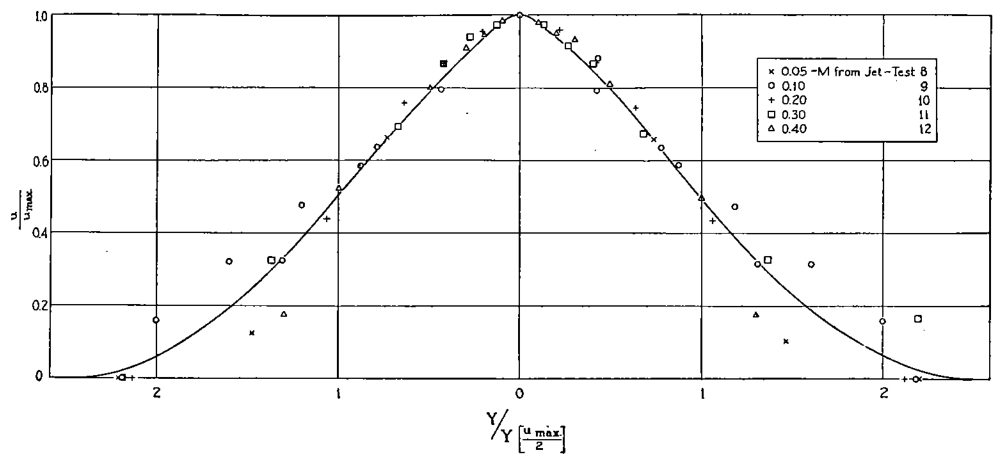

Fig. 4.—Non-dimensional representation of the observed distribution of axial velocities, according to measurements by Peters and Bicknell (the full line represents the theoretical distribution given by Tollmien).

of the wake stream and thus the vorticity does not vanish there. In the absence of frictional forces outside the wake stream the motion of the water drawn in along the edges should be very nearly irrotational. This discrepancy indicates that Tollmien's solution is only approximately correct. It follows from (50) and (42b) that the vorticity at the boundaries is given by

(51)
$$\zeta = -\frac{v}{2x},$$

and consequently the theoretically prescribed vorticity decreases rapidly downstream. The prescribed vorticity is cyclonic along the left edge, anticyclonic along the right.

It is now possible to determine the shape of the free surface of the wake stream in the rotating basin. The velocity distribution is, according to the previous reasoning, independent of the rotation of the system. The total pressure gradient is given by

(52)
$$\nabla p = \nabla p_c + \nabla p_r,$$

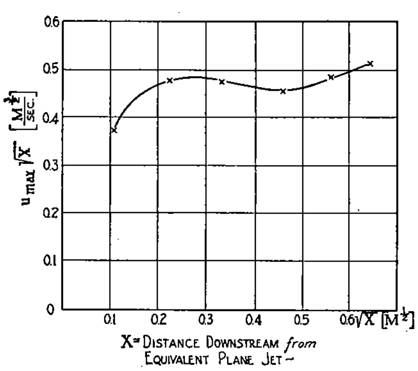

Fig. 5.—The product of axial maximum velocity and the square root of the distance from the jet as a function of the latter distance, according to measurements by Peters and Bicknell.

and since the residual pressure gradient is small it follows that the total pressure gradient very nearly agrees with the gradient of the Coriolian pressure field. If the elevation of the free surface above its equilibrium position be indicated by δ it follows that

$$\nabla p = \rho g \nabla \delta. \tag{53}$$

Thus

$$\frac{\partial p}{\partial x} = \rho g \frac{\partial \delta}{\partial x} = \rho f v = -\rho f \frac{\partial \psi}{\partial x} \tag{54}$$

and

$$\frac{\partial p}{\partial y} = \rho g \frac{\partial \delta}{\partial y} = -\rho f u = -\rho f \frac{\partial \psi}{\partial y}. \tag{55}$$

Consequently

$$\delta = -\frac{f}{g}\psi + \text{constant}. \tag{56}$$

Thus stream lines, isobars and lines of equal deformation of the free surface coincide.

Because of the prescribed inflow it follows that the water surrounding the wake stream cannot be at rest. Neglecting frictional forces outside the wake stream and considering the fact that the motion there is two-dimensional and, on account of the depth of the basin, very nearly free from horizontal divergence, it follows that the slow motion of the surrounding water must be very nearly irrotational.

Within the wake stream itself the deviations from geostrophic motion caused by the lateral shearing stresses are very nearly offset by the deviations due to inertia. The principal value of the preceding analysis lies in the establishment of the fact that the volume transport of a given current under the influence of lateral stresses must increase downstream. *Thus the wake stream, which appears to be a divergent current, is actually drawing in water from the surroundings.*

E. Wake Stream in Stratified Medium

The current system described in the preceding section is of limited interest only since it fails to take into consideration the stratification observed in the sea. It is an established fact that all well developed ocean currents are confined mainly to the troposphere. In the underlying stratosphere, which is separated from the troposphere by a transition zone of marked vertical stability, the observed motion is very sluggish. To some extent the effect of this stratification may be taken into account through the assumption that the basin is filled with two homogeneous, incompressible bodies of water; and that the motion, as a result of the stability of the internal boundary, is restricted to the upper layer.

An attempt will now be made to analyze the behaviour of a wake stream in such a basin. In spite of the simplifying assumptions introduced above, the system is too complicated to permit a detailed mathematical discussion and we are forced to restrict ourselves to a qualitative discussion of some of the principal characteristics of the motion.

The x-axis coincides with the axis of the current and points downstream, the y-axis points left. The density of the upper, lighter layer is ρ, that of the resting, lower and heavier layer is ρ'.

In accordance with the results of the preceding discussion it will be assumed that the momentum transport is constant although it will be found later that this assumption must be modified. It follows from (35) that the residual horizontal pressure gradient ∇_{pr} in the upper layer must be negligibly small. *The lines of constant deformation of the sea surface must, therefore, coincide with the isobars and stream lines in the upper layer.*

Let D be the actual thickness of the upper layer and D_0 the thickness of this layer in the undisturbed state in the absence of motion. Let K represent the depth of a certain fixed level in the lower, resting water layer ($K > D_0$). Then the pressure at this level is given by

$$p = \rho g D + \rho' g (K - D). \tag{57}$$

The depth K may be written

$$K = K_0 + \delta, \tag{58}$$

in which the expression K_0 is the depth of the level under consideration in the undisturbed case. Since there is no motion below the boundary, it follows that the horizontal pressure gradient at the level K must vanish. Thus

$$\rho g \nabla D + \rho' g \nabla (K - D) = 0 \tag{59}$$

and consequently

$$\nabla \delta = \frac{\rho' - \rho}{\rho'} \nabla D. \tag{60}$$

Combining (60) and (55) one finds

$$D - D_0 = \frac{\rho'}{\rho' - \rho} \delta = -\frac{f}{g} \frac{\rho'}{\rho' - \rho} \psi + \text{constant}. \tag{61}$$

Thus also the lines of constant depth of the internal boundary coincide with the stream lines.

The flow through a vertical section across the current system is obtained from

$$\rho f u = -\frac{\partial p}{\partial y} = -\rho g \frac{\partial \delta}{\partial y}, \tag{62}$$

or, through substitution of D for δ with the aid of (60),

$$\rho f u = -g \frac{\rho}{\rho'} (\rho' - \rho) \frac{\partial D}{\partial y}. \tag{63}$$

Since u is positive, the internal boundary between the two layers must dip down towards the right. Fig. 6 shows the general form of the internal boundary in a section across the current. In the computation of this diagram it was assumed that the velocity profile could be represented to a sufficient degree of accuracy through an equation of the form

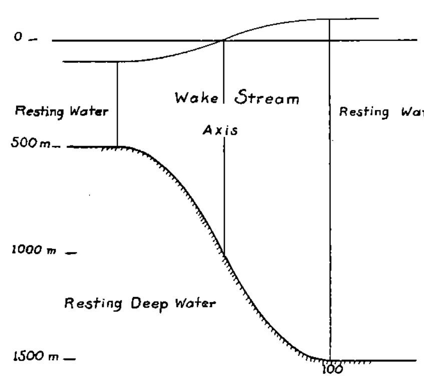

FIG. 6.—Theoretical cross-section through wake stream in stratified ocean under equilibrium conditions.

$$u = u_m \left(1 - \frac{y}{b}\right) \tag{64}$$

for $0 < y < b$, and

(65) $$u = u_m\left(1 + \frac{y}{b}\right)$$

for $-b < y < 0$. Thus $2b$ represents the width of the current. Fig. 2 and Fig. 4 indicate that the above assumption is reasonably correct.

The total mass transport (T) across the section is given by

(66) $$T = \int \rho u D dy = -\frac{g}{f}\frac{\rho}{\rho'}(\rho' - \rho)\int D\frac{\partial D}{\partial y}dy,$$

where the integration extends across the entire current width. This integral can be evaluated immediately without any knowledge of the details of the current profile, and represents a special application of a more general theorem first derived by Ekman[11] and later simplified by Werenskiold.[12] If D_l represents the depth of the upper layer at the left edge of the current and D_r the corresponding depth at the right edge, integration of the above expression for the mass transport gives

(67) $$T = \frac{1}{2}\frac{g}{f}\frac{\rho}{\rho'}(\rho' - \rho)(D_r^2 - D_l^2) = \frac{g}{f}\frac{\rho}{\rho'}(\rho' - \rho)D_m(D_r - D_l),$$

where

(68) $$D_m = \frac{D_r + D_l}{2}.$$

D_m represents the average depth of the upper layer across the section. In the absence of a pressure gradient directed along the axis of the current, D_m must remain fairly constant. *Thus the difference in level of the internal boundary between the two edges of the current must increase downstream as long as the mass transport increases in the same direction.*

It was stated above that the stream lines in Fig. 1 may be taken to represent the lines of equal depth D. An inspection of this diagram shows that the internal boundary gradually rises as one proceeds downstream along the left edge of the current while the reverse is true along the right edge of the current. It has now been shown that this result follows from the condition that the current must carry increasing amounts of water the further downstream the section is made and from the fact that the mass transport is proportional to the difference in depth of the upper layer on opposite sides of the current.

The increased mass transport downstream requires continuous inflow from the sides. The dynamically prescribed tilt of the internal boundary between the current and the resting deep water restricts the inflow on the left side of the current to a relatively narrow layer, whereas the inflow to the right of the current is spread out over a great depth. The transversal velocity must therefore be much greater along the left edge of the current than along the right. Because of the asymmetry of the current Tollmien's theory is no longer strictly applicable. However, as a first approximation it appears reasonable to assume that the speed of the inflowing water on the two sides is inversely proportional to the depth through which the inflow takes place. According to this assumption, the total inflow per unit length is the same on both sides of the current but may vary with the distance downstream.

Up to this point the current characteristics have been considered without any reference to conditions prevailing in the undisturbed water at some distance from the current. It is evident that this depth must depend upon factors largely unrelated to the current itself, such as the available supply and rate of formation of tropospheric water. Thus the values for D_l and D_r prescribed by the mass transport through a given section do not generally agree with the values of D prevailing at some distance to either side of the current. The water columns entering the wake stream must therefore undergo deformations which in turn produce vorticity in the surrounding water. The general character of these deformations will be discussed below.

It will be assumed that frictional forces can be neglected outside the wake stream. Because of the deformation of the individual fluid columns it is impossible to eliminate the deflecting force by means of the Coriolian pressure gradients (26, 27). It is therefore necessary to go back to the original equations of motion, which in this case take the form

$$\frac{du}{dt} = fv - g\frac{\partial \delta}{\partial x} \tag{69}$$

$$\frac{dv}{dt} = -fu - g\frac{\partial \delta}{\partial y}, \tag{70}$$

or, after substitution of D for δ with the aid of (60),

$$\frac{du}{dt} = fv - g\frac{\rho' - \rho}{\rho'}\frac{\partial D}{\partial x} \tag{71}$$

$$\frac{dv}{dt} = -fu - g\frac{\rho' - \rho}{\rho'}\frac{\partial D}{\partial y}. \tag{72}$$

The pressure gradients may be eliminated from these two equations through differentiation. Disregarding the terms multiplied with the vertical velocity component w in the expressions for the individual accelerations but permitting variations in latitude (f), we obtain a well known relation between vorticity (ζ) and horizontal divergence,

$$\frac{d(f+\zeta)}{dt} = -(f+\zeta)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right). \tag{73}$$

This equation has been used by Ekman[13] in an investigation of the motion over a corrugated ocean bottom and by J. Bjerknes[14] to explain the sinusoidal character of the flow of tropical air over a wavy polar front surface. It will be used below in a similar fashion.

Bjerknes 1932 p. 38 (not the complete eqn)

With the aid of the equation of continuity,

$$\frac{1}{D}\frac{dD}{dt} = -\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right), \tag{74}$$

the divergence may be eliminated from (73). Integration gives

$$f + \zeta = cD, \tag{75}$$

where c is a constant for any individual fluid column but may vary from one trajectory to another.

If we now consider an individual fluid column starting from zero vorticity at a great distance from the wake stream, where the depth of the column is D_0, it follows that

$$f = cD_0 \tag{76}$$

and consequently

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = f \frac{D - D_0}{D_0}. \tag{77}$$

In all attempts at a qualitative analysis of steady oceanic and atmospheric motions with the aid of the equation connecting vorticity and divergence it is well worth while to keep in mind that this relation must not be regarded as an adequate substitute for the equations of motion. An additional restriction on the motion is furnished by Bernoulli's equation, which in the present case of steady motion takes the form

$$\tfrac{1}{2}(u^2 + v^2) = k - g \frac{\rho' - \rho}{\rho'} D. \tag{78}$$

The constant k may vary from one stream line to another. It is important to remember that the relation between vorticity and divergence (77) is independent of the particular assumption of steady motion, which is indispensable in the derivation of Bernoulli's theorem.

A fluid column starting from rest and having the initial depth D_i will, as a result of the suction from the wake stream, reach the edge of the current with a velocity c given by

$$c^2 = \frac{2g(\rho' - \rho)}{\rho'} (D_i - D_e), \tag{79}$$

in which the expression D_e is the depth of the upper layer at the edge of the wake stream. It is obvious that D_i must be greater than D_e. *Consequently the columns shrink vertically as they approach the wake stream and this shrinking must be accompanied by an increasing anticyclonic vorticity.* It follows that the boundary between troposphere and stratosphere must be of the general character indicated in Fig. 7. This diagram shows that *the compensating movements set up in the surroundings of a wake stream must have a component in the direction of the current itself on the right side of the wake stream but must appear as a counter current on the left side.* The occurrence of such counter currents along the left edge of the Equatorial Current and of the Gulf Stream is well known.

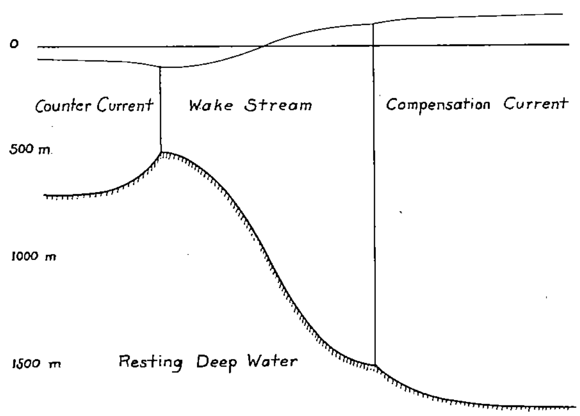

FIG. 7.—Theoretical cross-section through wake stream with fully developed counter and compensation currents.

The difference between the depth of the upper layer at the right edge of the current and the corresponding depth still further to the right in the undisturbed water must decrease downstream. *The compensation current to the right of the wake stream must therefore decrease in intensity downstream and finally become negligible.*

In this connection the merging of the Antilles Current with the Florida Current is of some interest. In the light of the theory outlined above, the Antilles Current must be regarded primarily as a compensation current resulting from the strong suction along the upstream section of the right edge of the Florida Current. In favor of this conception speaks the fact that the Antilles Current off the Bahamas is concentrated to a narrow ribbon of only 80 km. width, which is less than the width of the Florida Current.[15] Were it a branch of the North Equatorial Current one would expect it to appear as a broad band of low and uniform velocity. Iselin[16] has published temperature and salinity sections between Haiti and Bermuda which clearly indicate the continuation of the Equatorial Current through a uniform and very gradual slant of the isotherms and isohalines, but there is no sign of a concentrated current next to the continental shelf such as one observes further downstream, northeast of the Bahamas.

Returning to the theoretical cross-section through a wake stream represented in Fig. 7, it follows that the increase in volume transport downstream associated with the suction of the current produces an increased difference between the depth of the internal boundary at the left edge of the current and in the undisturbed water still further to the left. *Thus the volume transport of the counter current tends to increase downstream.**

If we assume that the columns drawn in along the left edge start out with approximately the same undisturbed depth, it follows from Bernoulli's equation that *the maximum velocity in the counter current must increase downstream*. The intense vertical shrinking to which these columns are subjected shows that they must possess strong anticyclonic vorticity; since the counter current must flow nearly parallel to the counter current we may conclude that *the velocity of the counter current reaches its maximum at or near the left edge of the wake stream and decreases rapidly with increasing distance from the latter.*

For rough estimates it is permissible to assume that the percentual difference in density between stratosphere and troposphere is of the order of magnitude 0.001. With this numercial value one finds

$$c^2 = 2(D_i - D_e),$$

indicating that a vertical shrinking of only 50 m. is sufficient to produce a velocity of 100 cm.p.s. This value is considerably in excess of observed counter current velocities. The apparent discrepancy is eliminated if the following two points are taken into consideration:

(a) The deep currents on the northern hemisphere move clockwise around the ocean basins and thus the amount of water enclosed between the left edge of a current and the nearest shelf is mostly fairly limited. The suction of the current itself is therefore generally capable of bringing about a reasonable degree of equilibrium between the dynamically prescribed depth D_l along the left edge and the undisturbed depth D_{0l} to the left of the current. This implies that D_{0l} is not quite constant but decreases somewhat downstream.

(b) Up to this point it has been assumed that frictional forces may be neglected outside the wake stream itself. It has been shown above that the horizontal stretching associated with the suction of the wake stream must produce strong vorticity, i.e. shearing motion, in the counter current. Lateral stresses must therefore develop also in the counter current which consequently assumes some of the characteristics of the wake stream itself. A consideration of Bernoulli's theorem indicates that the counter current at some point downstream must reach such an intensity that its suction reverses the normal direction of mass transfer between the two current systems. *Water will then be ejected from the wake stream into the counter current.* In the counter current this water is mixed with water drawn in from the undisturbed layers to the left of the system. Through this process both

* The expressions downstream and upstream refer to the main current.

the anticyclonic vorticity and the velocity of the counter current are reduced. The discharge of water from the wake stream has the effect of restoring its mass transport to an amount more nearly in equilibrium with the undisturbed depths of the upper layer on each side of the current system.

It was brought out above that the absorption upstream must take place primarily along the right edge. *Through this upstream absorption along the right edge combined with the downstream discharge of eddies along the left edge, water may be transferred across the current from the open ocean basin to the right into the limited body of water to the left of the current.*

In Tollmien's theory the water absorbed from the surroundings possesses no momentum in the direction of the current. We have seen that the absorption along the left edge of the wake stream necessarily leads to the development of a strong counter current. The water absorbed from this counter current into the wake stream possesses a certain momentum directed upstream. Thus, *due to the absorption of water from the counter current the momentum transport through a section normal to the axis of the current cannot remain constant but must eventually decrease downstream.* Furthermore, since the water absorbed along the left edge of the current possesses momentum upstream we are justified in saying that the innermost part of the counter current belongs to the wake stream proper. The highest point of the internal boundary will no longer be found at the left edge of the current but well inside the wake stream itself.

Up to the present point it has been assumed that the lower layer does not participate in the motion. This cannot be strictly true. It has just been shown that the internal boundary tends to form a dome separating the wake stream from its counter current. It is reasonable to assume that normal and lateral stresses will develop along the internal boundary in this region and that stratosphere water will be drawn into the wake stream and into the counter current along the slopes of this dome as a result of the suction exerted by these two currents. Thus a certain amount of "upwelling" along the left edge of the current must result from the wake stream mechanism.

In the axis of the current the internal boundary reaches its maximum tilt. Vertical water columns transferred laterally will here tend to intersect the internal boundary and thus momentum will eventually be communicated also to the stratosphere as a result of lateral shearing forces. This gradual transfer of momentum to deeper and deeper strata is clearly indicated by observations from the Gulf Stream system.

In the language of classical hydrodynamics the mechanism outline above may be described by saying that a *terrestrial wake stream in a stratified medium acts like a series of sinks with respect to the surrounding medium. Large quantities of fluid must be removed from the surroundings and this removal is associated with the creation of a counter current of strong anticyclonic vorticity along the left edge. The mass transport increases downstream but is intermittently restored to a prescribed value through the discharge of eddies along the left boundary of the current.*

The analogy between wake streams and ocean currents presented above implies that the currents of troposphere must not be regarded as "rivers" flowing through resting basins. The water flowing through any one section rapidly loses its identity through turbulent exchange with the surrounding medium.

It is tempting to apply the preceding reasoning to certain types of atmospheric flow, particularly to the formation of tropopause funnels[17] along the left edge of polar air outbreaks, but this problem is far more complicated on account of the nonstationary character of the atmospheric systems. The author hopes to be able to return to this problem on a later occasion.

II. APPLICATIONS TO THE GULF STREAM SYSTEM

Selected hydrographic observations from the Gulf Stream system will be analyzed below in a preliminary attempt to trace the actual exchange of water between the current and its surroundings. *Atlantis* observations only will be used; the positions of the individual stations are indicated on Chart I. Velocity distributions, mass and momentum transport through various sections across the Gulf Stream will be discussed in a separate contribution.

It was stated above that the tilting of the isopycnic surfaces in an oceanic wake

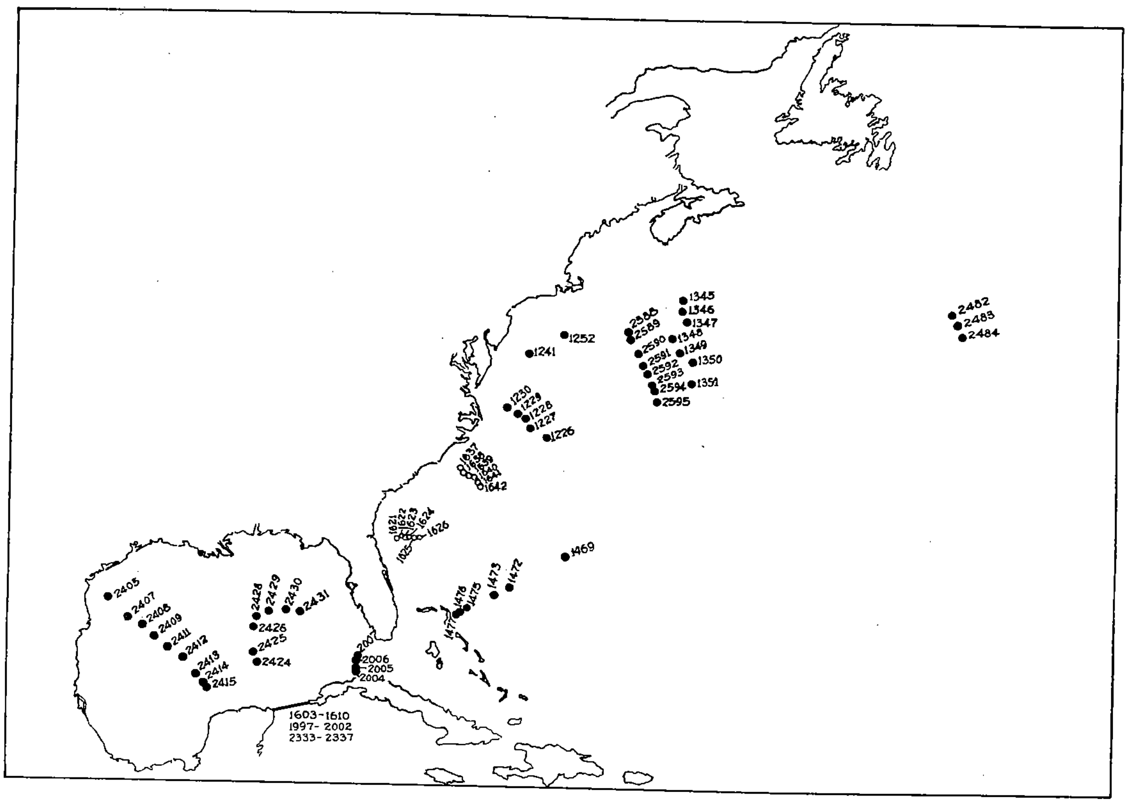

Chart I.—Locations of stations used.

stream is the result, rather than the cause of the motion. In a resting ocean basin, isothermal and isohaline surfaces are horizontal and consequently there exists a well defined correlation between temperature and salinity, constant from station to station. If the equilibrium is upset through the piling up of wind-driven surface water in certain regions, the resulting pressure gradients will be transmitted downward and set the subsurface layers in motion. A banking of the individual strata results but this banking does not affect the correlation between temperature and salinity. Lateral mixing may gradually alter the temperature-salinity correlation, but the rate of change must be extremely slow since the eddies, particularly in regions of marked vertical stability, are forced to move along isopycnic surfaces.

Exceptions to the rule of constant temperature-salinity correlation across the current should occur primarily in those regions where one or the other of the two basins separated by the current is so limited in size that its water may be modified by admixture of coastal water. In the northern hemisphere, deep currents tend to move clockwise around the ocean basins and the amount of water enclosed between the left edge of a current and the continental shelf is therefore often fairly limited. In this case one

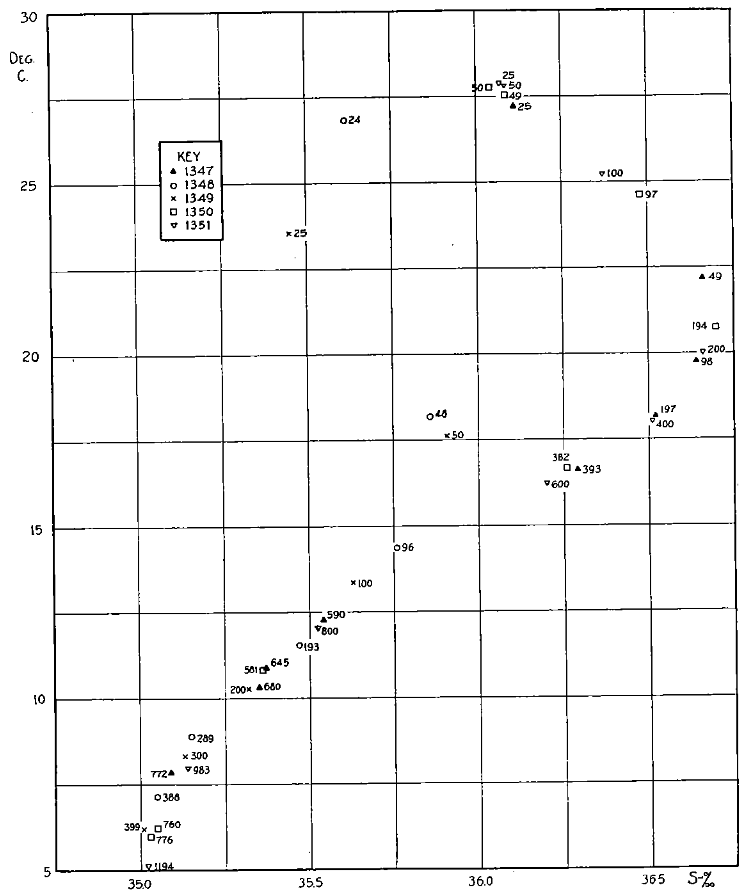

Fig. 8.—Temperature-salinity correlation curves for Nova Scotia section (Atlantis stations 1347–1351). The number beside each observation gives its depth in meters.

should expect to find anomalies in the temperature-salinity correlation in the left half of the current, where fresher water is absorbed.

F. Temperature-Salinity Correlations

Temperature-salinity correlations from five stations in a section across the Gulf Stream between Nova Scotia and Bermuda are plotted in Fig. 8. Station 1350 is located very near the axis, while 1349 is on the northern and 1351 on the southern edge of the current. It is seen that the correlation is remarkably constant across the system with the exception of four points within the first fifty meters of the surface north of the current system (stations 1348 and 1349).

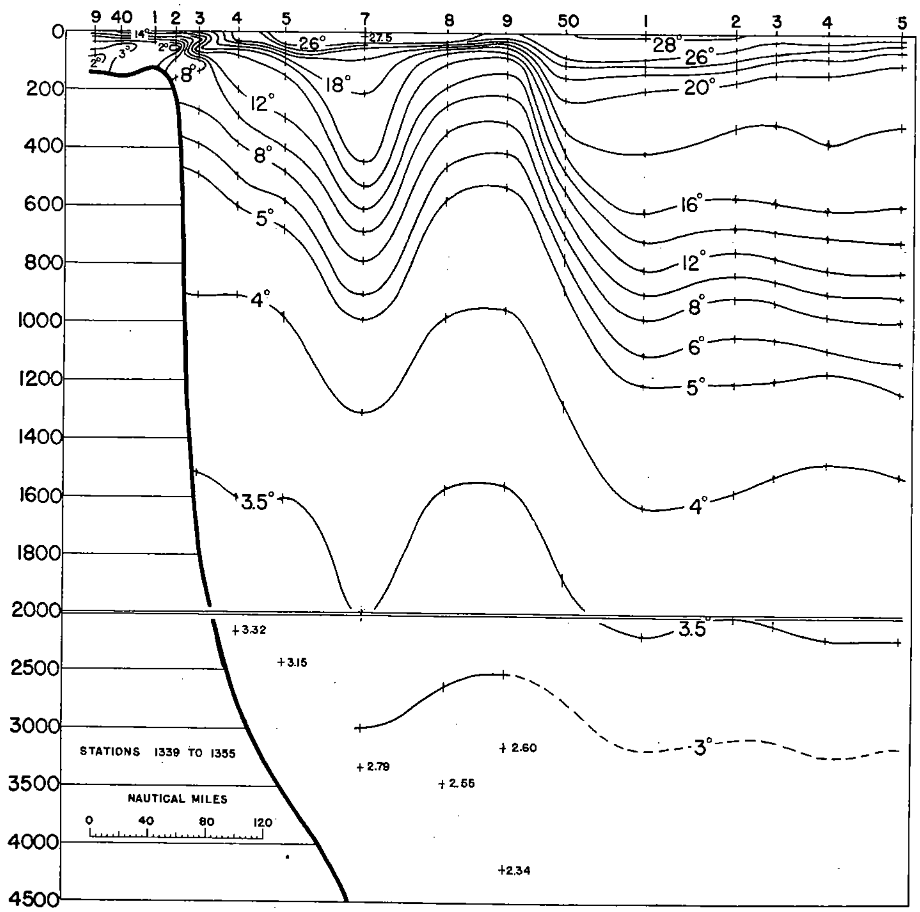

FIG. 9.—Temperature distribution in Nova Scotia section, according to Iselin.

Fig. 9, which was placed at my disposal by Mr. Iselin, shows the vertical temperature distribution in the same section. Station 1347 is located in the center of what appears to be a well-developed anticyclonic eddy to the left of the current. The temperature-salinity correlation for this station indicates that the central portion of the "eddy" consists of pure Gulf Stream water. The values of temperature and salinity in the surface layer at stations 1348 and 1349 may perhaps be interpreted as representing water which has been freshened through admixture of coastal water and which is now about to be absorbed by the current after having been cut off from its source by an arm of pure Gulf Stream water.

Fig. 10 shows the temperature-salinity correlations from the stations in a section between Chesapeake Bay and Bermuda, where the current runs fairly close to the continental slope. Station 1228 is near the axis of the current. The observations from station 1230, which is to the left of the current, and 1229, which is located inside the current but near its left edge, show admixture of fresher water in the surface layers. At station 1228, in the middle of the current, this anomaly extends to a depth of somewhat more than 200 m. The tendency of the freshening effect to spread downward from left to right may

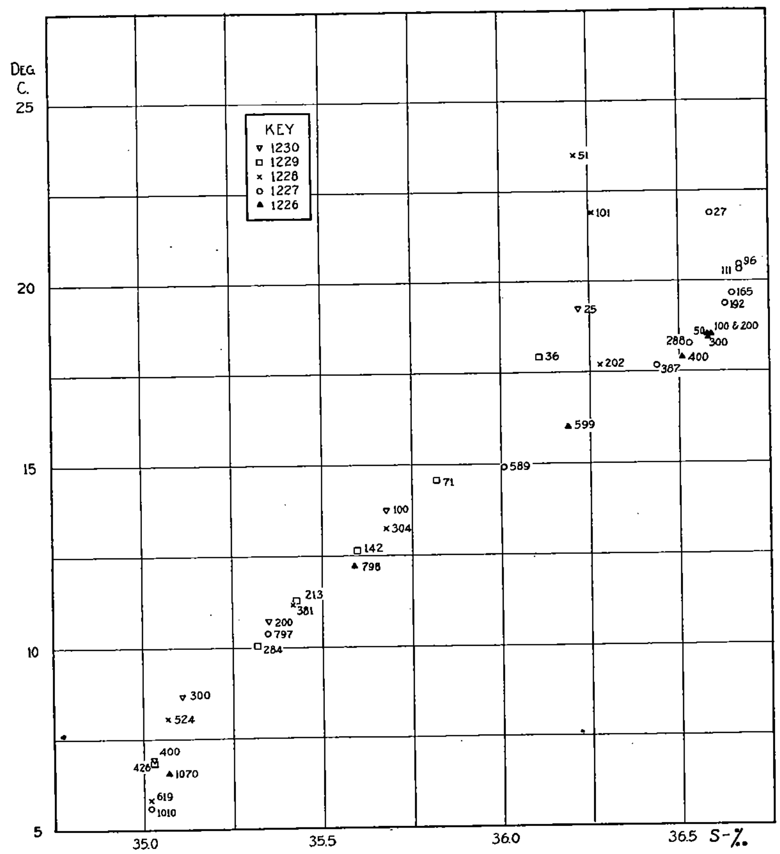

FIG. 10.—Temperature-salinity correlation curves for Chesapeake Bay section (Atlantis stations 1226–1230).

perhaps be explained as the result of a lateral movement of the absorbed water along the surfaces of constant density.

The curves from the Onslow Bay section (Fig. 11) display similar characteristics. The surface layers in the left half of the current show the effect of coastal waters down to a depth of about 200 m. Station 1642 to the right of the current is characterized by surprisingly fresh water in the uppermost layer, but otherwise the temperature-salinity correlation is remarkably constant.

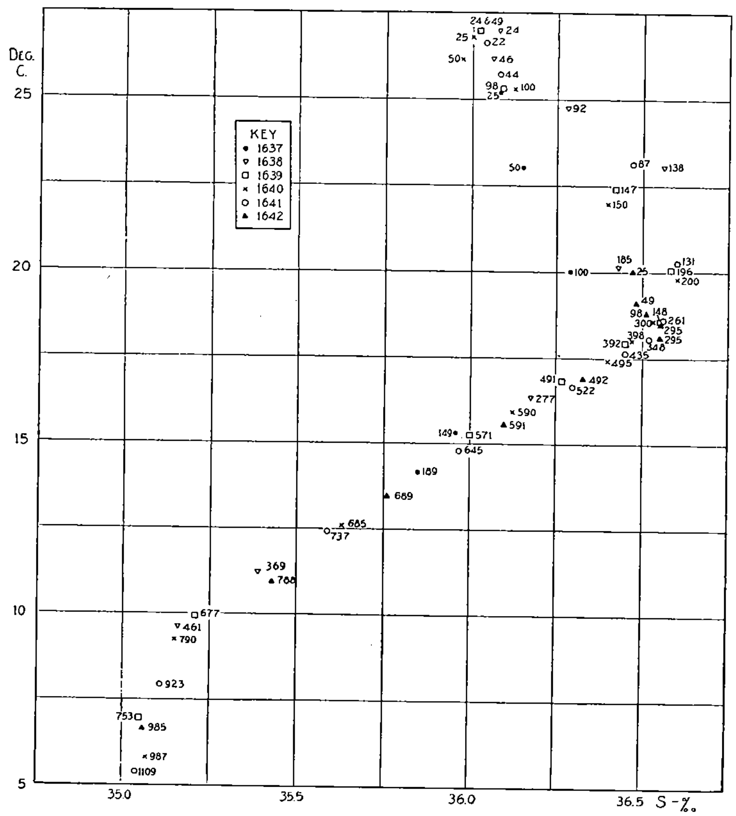

FIG. 11.—Temperature-salinity correlation curves for Onslow Bay section (Atlantis stations 1637–1642).

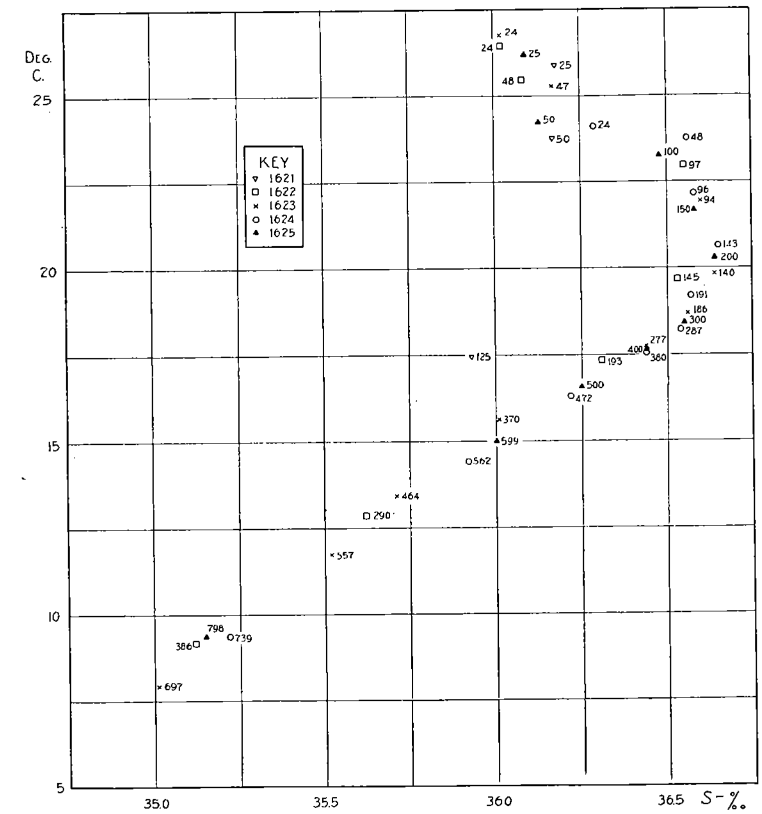

FIG. 12.—Temperature-salinity correlation curves for the Jacksonville section (Atlantis stations 1621–1625).

The curves from a section through the Florida Current off Jacksonville are given in Fig. 12. Station 1621 was made in shallow water inside the current and clearly exhibits admixture of coastal water, but otherwise the correlation is fairly constant.

As a last illustration we shall discuss the temperature-salinity correlations at three stations in the Gulf Stream south of the Grand Banks, (Fig. 13). Unfortunately the section does not extend all the way across the current, the southernmost station (2484) being south of the axis although well within the current itself. In this region the current comes in close contact with water which has been freshened, directly or indirectly, through admixture of Arctic water. The absorption of this water by the current is quite readily seen. At station 2484 the maximum salinity is somewhat lower than further upstream, indicating that the freshening effect extends across the axis of the current.

This brief examination seems to indicate that the Gulf Stream between Jacksonville and the Grand Banks is characterized by a fairly constant temperature-salinity correlation. Anomalies occur where the current flows so close to the continental shelf that the

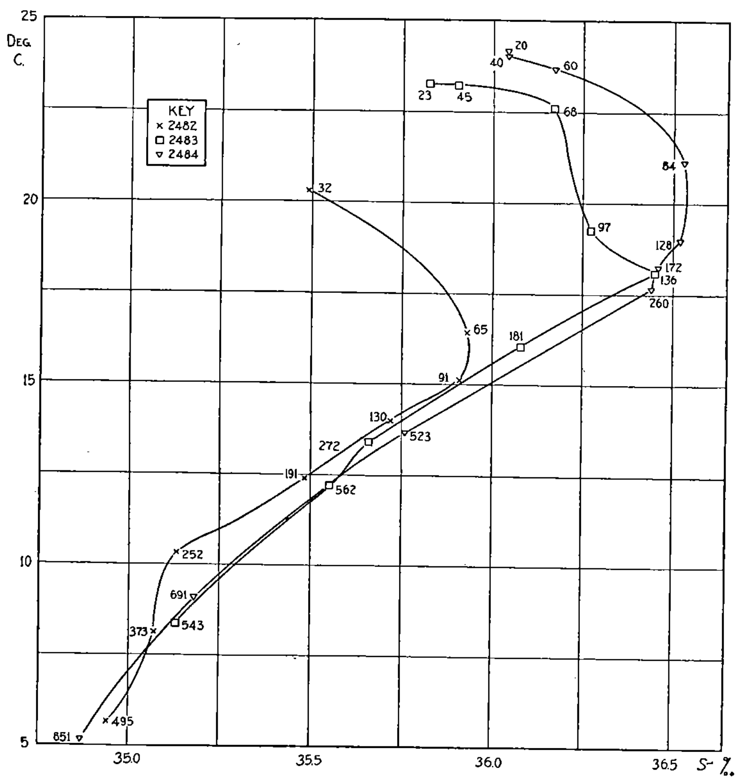

Fig. 13.—Temperature-salinity correlation curves for the Grand Banks section.

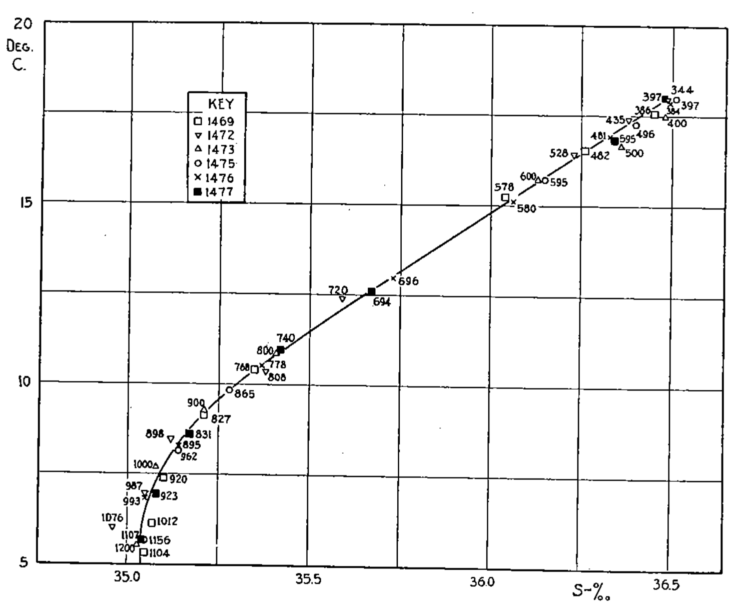

Fig. 14.—Average temperature-salinity curve for selected Sargasso Sea stations

limited body of water on its left becomes appreciably modified in the surface layers through admixture of coastal or northerly water masses. In these cases the observations indicate absorption of fresher water by the current system.

Before leaving the temperature-salinity correlations, we shall discuss briefly the effect of vertical mixing. Every section investigated exhibits a well marked salinity maximum. In the Jacksonville section, station 1623, near the axis of the current, has a maximum salinity of 36.64°/₀₀ at a depth of 140 m. At station 1640 in the Onslow Bay section the salinity reaches a maximum value of 36.60°/₀₀ at a depth of about 200 m. At Chesapeake Bay the central station (1228) is affected by coastal water, but at station 1350 in the Nova Scotia section the maximum is 36.70°/₀₀ at 194 m. If vertical mixing alone controlled the salinity distribution, this maximum would decrease downstream. The observations listed above show that it is maintained unimpaired from the Straits of Florida to Nova Scotia, which is a good indication of steady horizontal absorption of Sargasso Sea water. It was brought out in the theoretical analysis that the absorption along the right edge of the current must diminish downstream, while the intensity of the intermittent absorption and discharge along the left edge must increase in the same direction. This conclusion is in good agreement with the decrease of the salinity maximum observed between the Nova Scotia section discussed above and the Grand Banks section, where the maximum at station 2484 somewhat south of the current axis drops to about 36.55°/₀₀.

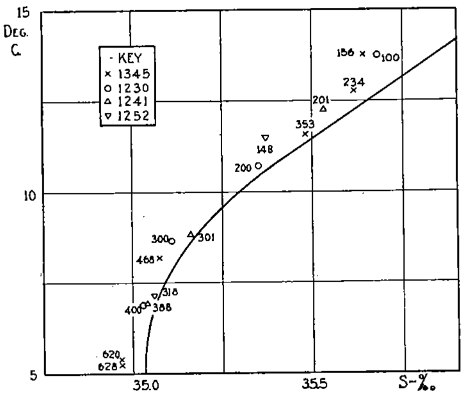

FIG. 15.—Average temperature-salinity correlation for selected slope water stations (full line represents average Sargasso Sea correlation).

The temperature-salinity correlations from five selected stations in the Sargasso Sea are plotted in Fig. 14. The curve obtained from this diagram has been transferred to Fig. 15, which also contains the temperature-salinity correlations from four selected stations in the slope water basin north and west of the Gulf Stream between Chesapeake Bay and Nova Scotia. Points above 100 m. have been excluded. If there were no communication between the two bodies of water separated by the Gulf Stream, the steady addition of fresh water to the basin would eventually produce marked differences between the two mean correlations. Actually, there is a fairly good agreement between the two sets of data, showing that coastal and northerly waters play a minor rôle in the production of slope water.

G. Oxygen-Salinity Correlations

The preceding brief examination shows that temperature and salinity are too conservative to be of much value in our attempt to trace the mass exchange between the current and its surroundings. Below a depth of one or two hundred meters an individual layer may retain its temperature and salinity for indefinite periods of time, particularly if the water is at rest. On the other hand, *determination of the intensity of the horizontal exchange between the current and its surroundings requires an indicator which changes only slightly during the comparatively short time needed by the water to travel from one current* *section to the next but which does undergo considerable variations during the longer time intervals spent by the water at rest within the "source regions" outside the current itself.*

These requirements are fairly well satisfied by the oxygen content of the water below a depth of about 200 m. The oxygen content of the deep water is replenished through chilling, mixing and sinking of surface water in the polar regions. Between the oxygen-rich surface and bottom layers the oxidation of sinking organic material produces a layer of minimum oxygen content. In regions of stagnation, the reduction in the oxygen content presumably continues until the horizontal and vertical oxygen gradients become sufficiently steep to insure adequate compensation, through turbulent transport, for the losses caused by oxidation *in situ*. The resulting equilibrium may vary from one source region to another, depending upon its biological and hydrographic characteristics.

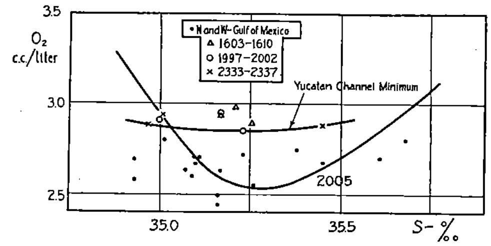

FIG. 16.—Comparison between minimum oxygen content of water passing through the Yucatan Channel and the minimum oxygen content in the Havana section (station 2005) and at stations in the northern and western parts of the Gulf of Mexico.

It is probable that a redistribution of oxygen must occur along the continental slopes where the prevailing circulation causes a banking and probably also some upwelling of deep water. In these regions, turbulence originating on the slopes produces lateral mixing with a component across the isopycnic surfaces and thereby also a transfer of oxygen from the bottom layer to intermediate strata.

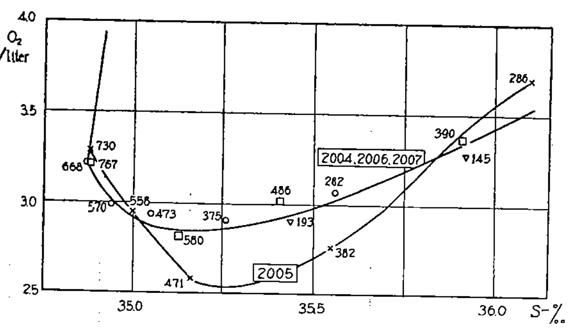

FIG. 17.—Oxygen-salinity correlation curves for the Havana section.

Seiwell's investigations[18] indicate that the order of magnitude of the oxygen consumption in the minimum layer in the Sargasso Sea is about 0.1 cc. per liter and year, but the basis of this estimate has recently been questioned. A dependable estimate of the oxygen consumption has been made by Redfield from a study of the decrease of oxygen in a stagnant pool of water below the sill depth of the Gulf of Maine. The estimated consumption below 200 m. is about 0.36 cc. per liter and year.* There is some reason to assume that the consumption is less in the deep ocean basins and it would therefore seem permissible to assume that the oxygen content can be regarded as a conservative property for the time interval required by water of average Gulf Stream velocity to travel from the Straits of Florida to the Grand Banks (less than three months).

Three sections are available from which the oxygen content of the water entering the Gulf of Mexico through the Yucatan Channel may be determined (stations 1603–1610, 1997–2002, 2333–2337). For these stations, all minimum oxygen values of less than 3.00 cc. per liter have been plotted against salinity and a smooth curve drawn to indicate the probable minimum oxygen content of the water entering the Gulf of Mexico through the Yucatan Channel (Fig. 16).

Fig. 17 represents the oxygen-salinity curves for the individual stations in a section

* The author is indebted to Dr. A. C. Redfield for this information, which has not yet been published.

across the Florida Current slightly west of Havana. Station 2005, which is close to the axis of the current, exhibits an oxygen minimum which is considerably below the lowest value recorded in the Yucatan Channel. The curve for this station has been transferred to Fig. 16 to bring out the discrepancy more clearly. In the same figure are entered all minimum oxygen values below 2.80 cc. per liter recorded at various stations in the western and northern part of the Gulf of Mexico. The absolute minimum is 2.44 cc. at station 2413 in the western half of the Gulf, while the minimum at station 2005 in the Havana section is 2.58 cc. per liter. The minimum oxygen content increases on both sides of the axis in the Havana section, probably on account of strong horizontal stirring along the boundaries of the channel.

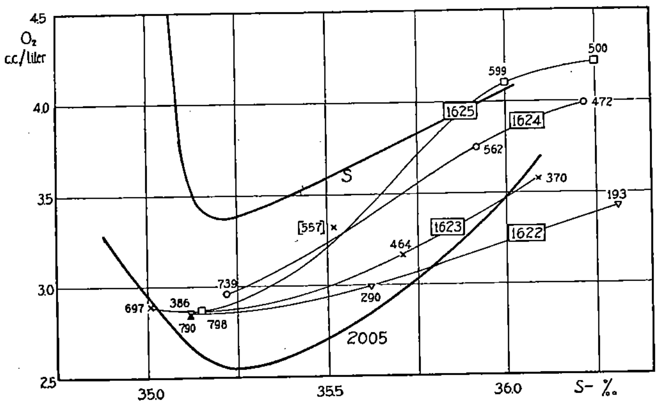

FIG. 18.—Oxygen-salinity correlation curves for the Jacksonville section.

It seems reasonable to conclude that the deep water entering the Gulf of Mexico through the Yucatan Channel largely loses its identity before it has an opportunity to leave the Gulf through the Straits of Florida. This is more readily understood if one considers the fact that the current must bend sharply to the right in the Gulf and that the convex edge of a curved current is dynamically unstable. It is probable that the deep water current from the Yucatan Channel breaks up in large eddies in the Gulf and then re-establishes itself near the western entrance to the Straits of Florida. Support for this point of view may be found in one of Iselin's diagrams (l. c. Fig. 47), which shows the topography of the 10° isotherm in the Gulf of Mexico. The diagram clearly indicates the existence, in the eastern part of the Gulf, of a large clockwise eddy which necessarily blocks the direct transfer of deep water from the Yucatan Channel to the Florida Straits.

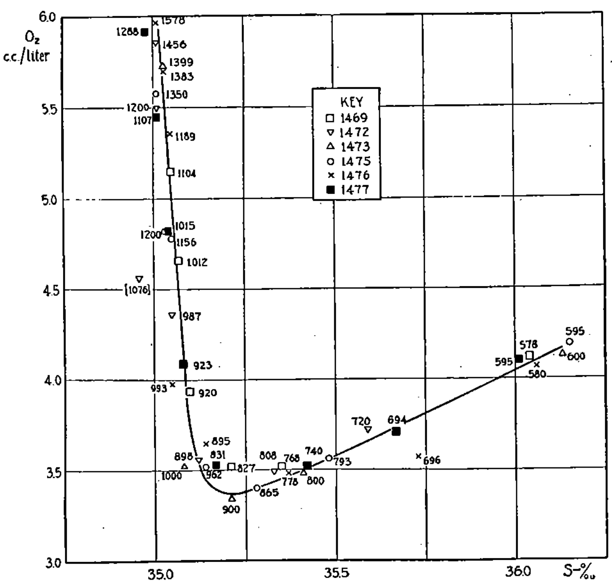

FIG. 19.—Average oxygen-salinity correlation curve for selected Sargasso Sea stations.

The extremely low minimum oxygen content in the central portion of the Havana section cannot be traced through the Florida Straits. The current is here forced through a narrow passage at increased speed; lateral mixing originating at the boundaries of this channel rapidly brings about a uniform distribution of the oxygen content. This mixing is well illustrated by the fairly small spread of the oxygen-salinity curves for the Jacksonville section (stations 1621–1626) in Fig. 18.

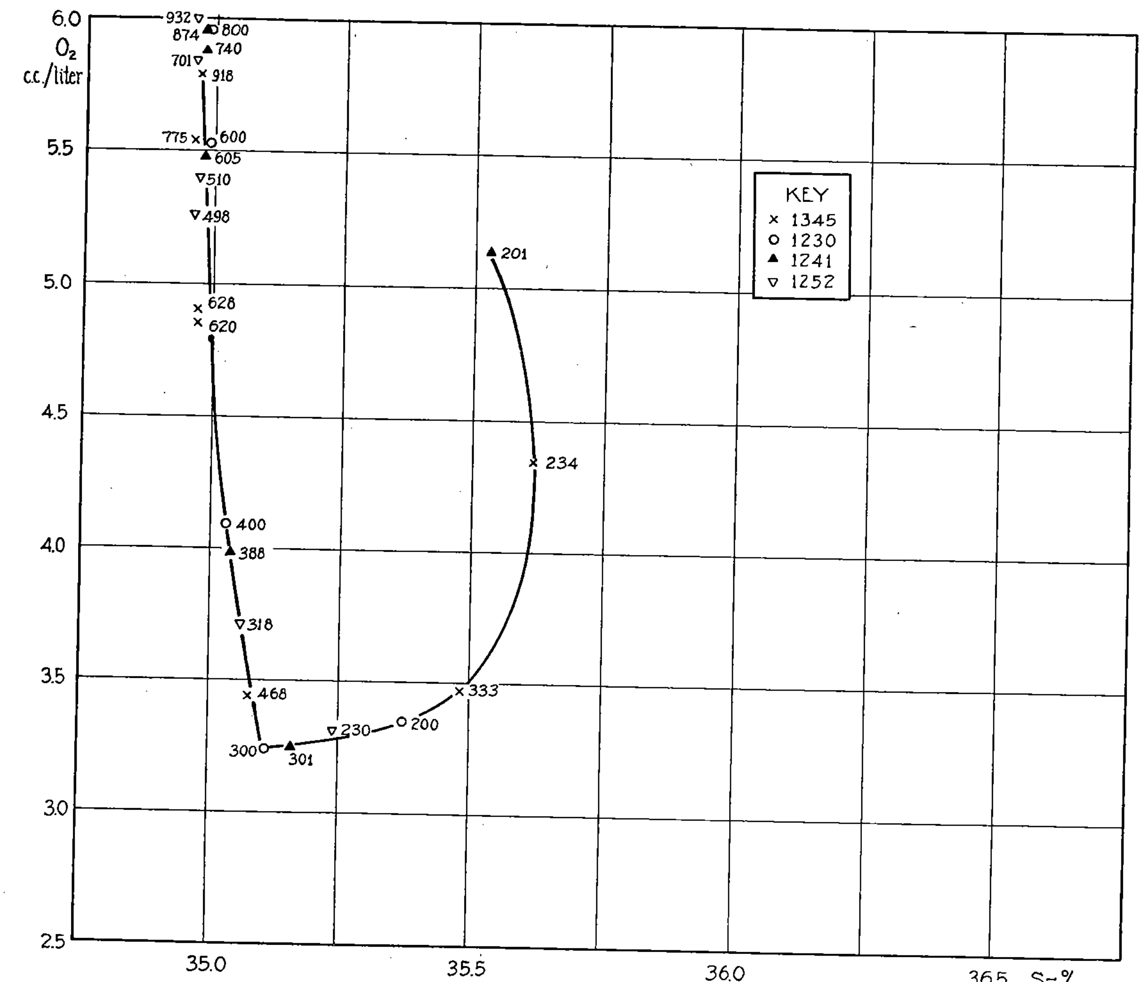

FIG. 20.—Average oxygen-salinity correlation curve for selected slope water stations.

For purposes of comparison, two reference curves have been reproduced in Fig. 18, one being the oxygen curve for station 2005 in the Havana section and the other a mean oxygen-salinity correlation curve for five selected Sargasso Sea stations on a line between Nassau and Bermuda. Fig. 19 contains the individual values on which this latter reference curve is based.* A third reference curve, Fig. 20, represents the average oxygen-salinity correlation obtained from four selected stations in the slope water basin between Cape Hatteras and the Grand Banks. With the aid of these curves the gradual mixing of the Florida Current with its surroundings may be traced.

* Note that the stations used all lie in the southwestern part of the Sargasso Sea. It will be shown below (last page) that the northwestern Sargasso Sea has higher minimum oxygen values, and hence does not fit this characteristic curve exactly.

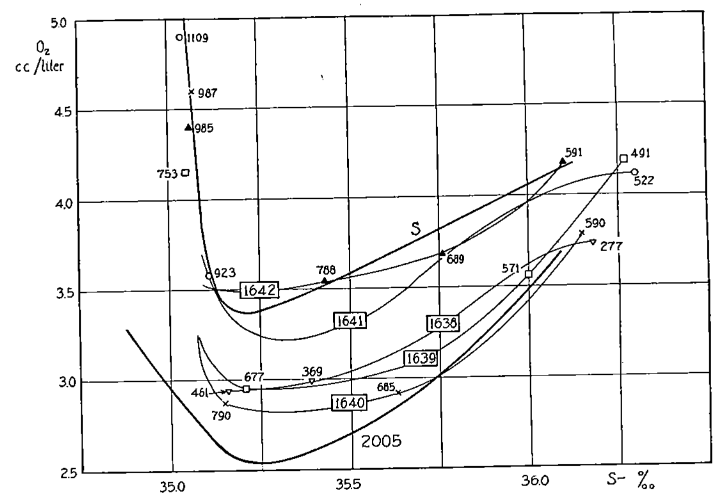

FIG. 21.—Oxygen-salinity correlation curves for the Onslow Bay section.

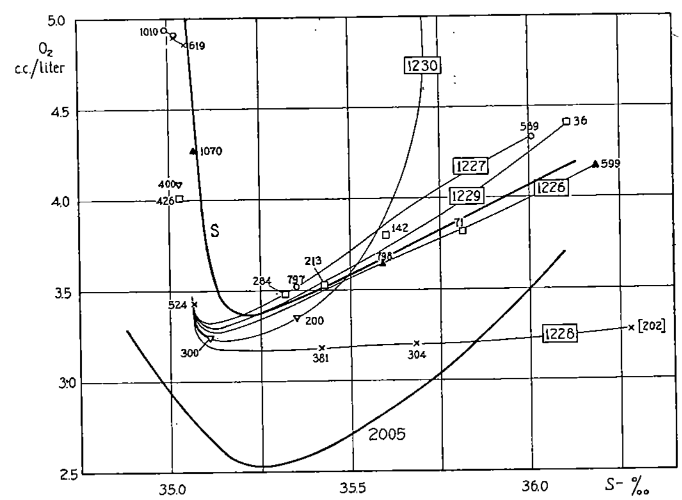

FIG. 22.—Oxygen-salinity correlation curves for the Chesapeake Bay section.

The oxygen-salinity curves for the Onslow Bay section (Fig. 21) are spread out in a broad band, the outer stations having a higher oxygen content than the ones closer to the shore which more nearly agree with the Jacksonville curves. Thus strong absorption must occur along the right edge of the current on the stretch between Jacksonville and Onslow Bay.

The Chesapeake Bay curves are very instructive. By comparing the group of curves in Fig. 22 with the curves from the Jacksonville section (Fig. 18) it appears that the oxygen content everywhere has risen, indicating absorption of Sargasso Sea water to the right and of slope water to the left.* One station only, 1228 near the axis of the current, shows a decidedly lower oxygen content, although in excess of the value observed in the Jacksonville section.

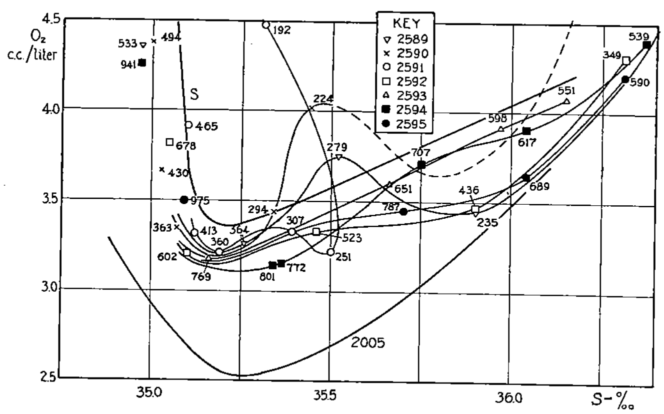

FIG. 23.—Oxygen-salinity correlation curves for the Georges Bank section.

Beyond Chesapeake Bay, absorption along the right edge apparently decreases and must finally give way to a discharge of Gulf Stream water into the Sargasso Sea. Along the left edge, the current breaks up in eddies, produced by intermittent absorption of slope water and discharge of Gulf Stream water into the slope water basin. It will be shown in a separate paper, now under preparation, that the mass transport decreases somewhat between Chesapeake Bay and Nova Scotia and that consequently water must be ejected from the Gulf Stream on this lap. The continued horizontal mixing must lead to an equalization of the oxygen distribution in the remaining central part of the current. The resulting distribution ought to be enclosed between the extremes observed in the Chesapeake Bay section. This equalization of the oxygen distribution is well illustrated by the oxygen-salinity curves from a recent section across the Gulf Stream south of Georges Bank (Fig. 23).

* The increase of the oxygen content in the minimum layer on the left edge of the current may also be due to horizontal mixing across the isopycnic surfaces next to the continental slope.

Fig. 24 represents the temperature distribution in the same section. A well developed eddy of the type previously found in the Nova Scotia section is clearly indicated. The oxygen-salinity curve for station 2591 located in the dome on the left side of the current agrees fairly well with the typical slope water correlation curve in Fig. 20, but

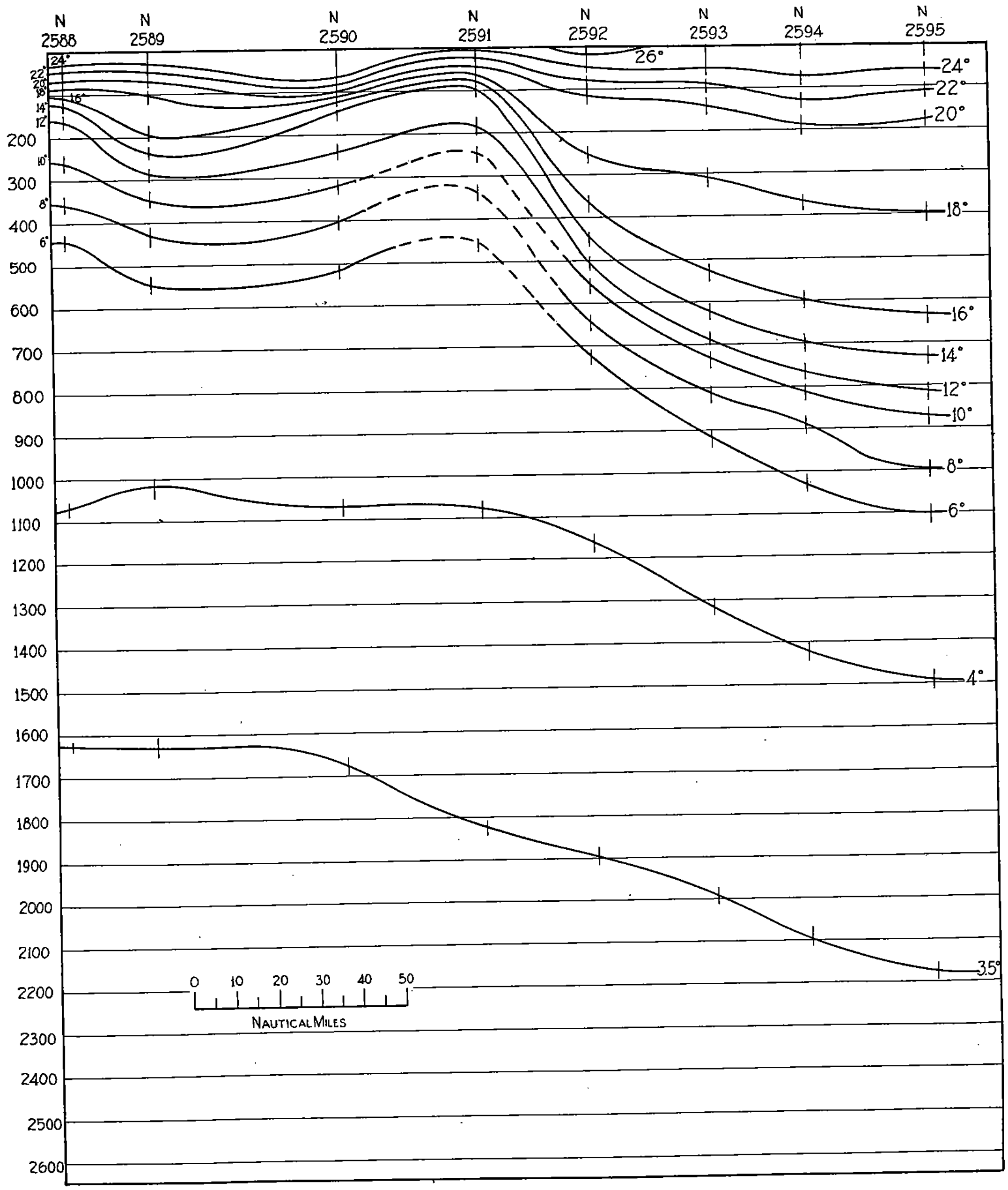

FIG. 24.—Temperature distribution in the Georges Bank section.

north and south of this dome the oxygen-salinity curves are fairly uniform and enclosed between the two extremes indicated by the curves from the Chesapeake Bay section in Fig. 22. Fig. 25 is intended further to illustrate the Gulf Stream characteristics of the water at station 1347 in the central part of the eddy in the Nova Scotia section reproduced in Fig. 9.

In a recent article Dietrich[19] expresses the opinion that since the oxygen content of the surrounding water masses is everywhere higher, the oxygen minimum layer observed in the Chesapeake Bay section must be the result of oxidation *in situ*. According to Dietrich, this oxygen minimum would be erased by vertical turbulent transfer from the surrounding oxygen-richer layers if the water were in motion. He concludes that the minimum layer, formed by oxidation *in situ*, must be at rest and bases his calculations of the velocity distribution in the Chesapeake Bay section (Atlantis stations 1226–1930) on this assumption.

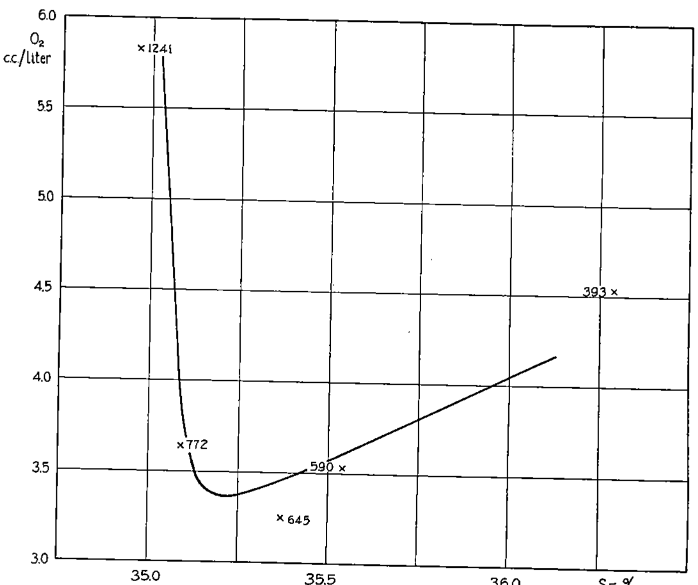

FIG. 25.—Comparison between the oxygen-salinity correlation in the Gulf Stream eddy (station 1347) with the typical Sargasso Sea relationship.

Against this interpretation several objections may be raised. It ignores the fact that the lowest oxygen values are found in the Florida Straits and that the minimum off Chesapeake Bay thus may be the result of advection. Iselin (l.c. Fig. 42) has shown, with the aid of temperature-salinity correlations, that the bottom water off Cape Canaveral on the east coast of Florida is identical with the deep water in the Yucatan Channel and distinct from typical Sargasso Sea water, indicating that there must be some motion eastward through the narrow part of the Florida Straits in the oxygen-minimum layer which here is close to the bottom.

Dietrich's interpretation does not explain why the oxygen-minimum layer is most well marked in the current itself and why this minimum becomes increasingly well marked as one proceeds upstream. Finally, the assumption that the oxygen layer may be regarded as a zero surface for the velocity distribution by necessity implies that there is a fairly strong motion upstream below the minimum layer. In Fig. 26 we have reproduced the topography of the individual isobaric surfaces as computed by Dietrich. The diagram shows that the 1000 decibar surface drops about 24 dyn. cm. from left to right between stations 1229 and 1227. The distance between these two stations is about 93 km. It follows that the mean velocity upstream at a depth of 1000 m. must be about 28 cm.p.s. Iselin's data show that the temperature is consistently slightly lower at station 1229 than at station 1227 also for depths below 1000 m. (l.c. Fig. 5). From this fact and from the well-established constancy of the temperature-salinity correlation across the section it is evident that the water column below the 1000 m. level at station 1229 is heavier than the corresponding column below station 1227. It follows that the mean velocity upstream between these two stations at all levels below 1000 m. must be in excess of 28 cm.p.s. Since the mean depth of the section is about 4000 m., Dietrich's assumptions imply that the amount flowing upstream below the 1000 m. level must be in excess of $78 \cdot 10^6$m.^{3}/sec., or more than double the $31 \cdot 10^6$m.^{3}/sec. carried downstream according to the same calculation. Dietrich's value for the transport agrees well with Wüst's estimate that the Florida Current and the Antilles Current jointly carry about 35.5 m.^{3}/sec., but this agreement is more than offset by the intense and wholly unexplainable deep water counter current implied by his assumptions. Furthermore, it is impossible to explain away this counter current by saying that frictional forces in the practically homogeneous deep water prevent the development of a current corresponding to the indicated slope of the 1000 decibar surface. There is every reason to believe that the normal stresses originating at the bottom must vanish within a very short distance of the bottom. Evidence to support this viewpoint may be found in the uniformly low *Austausch* values obtained by various investigators for the deep ocean basins. Lateral stresses may be of significance in the deep water but these stresses, which act in the direction of the movement, do not affect the balance of forces normal to the axis of the current, from which balance all current velocities are calculated.

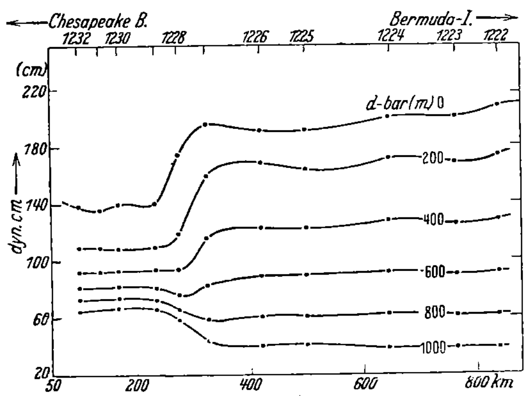

FIG. 26.—Topography of individual isobaric surfaces in the Chesapeake Bay section, according to Dietrich.

Wüst's studies[20] indicate that there must be a slow movement southward of the oxygen-minimum layer in the western half of the North Atlantic but there is no reason to assume that this motion is concentrated below the Gulf Stream as indicated by Dietrich. Iselin suggests, on the basis of the average temperature distribution in a section between Chesapeake Bay and Bermuda (l.c. Fig. 24), that this slow southward drift occurs considerably east of the Gulf Stream, in the vicinity of Bermuda.

Iselin has published a velocity distribution for the same section computed on the assumption that the speed of the water is negligible at a depth of 2000 m. (l.c. Fig. 27). Because of the practically homogeneous character of the water below 2000 m. this assumption eliminates upstream movements in the deep water. The computation indicates that the vertical rate of shear near the axis of the current (between stations 1227 and 1228) is practically constant from a depth of 300 m. down to about 950 m. (120 cm.p.s. at

300 m., 10 cm.p.s. at 950 m.). Since the value for the rate of shear is fairly independent of the choice of zero level for the velocity, it follows that there must be a considerable rate of shear, and thus some vertical turbulent transfer, at the oxygen-minimum level in the axis of the current. The vertical rate of shear in the minimum layer decreases toward both

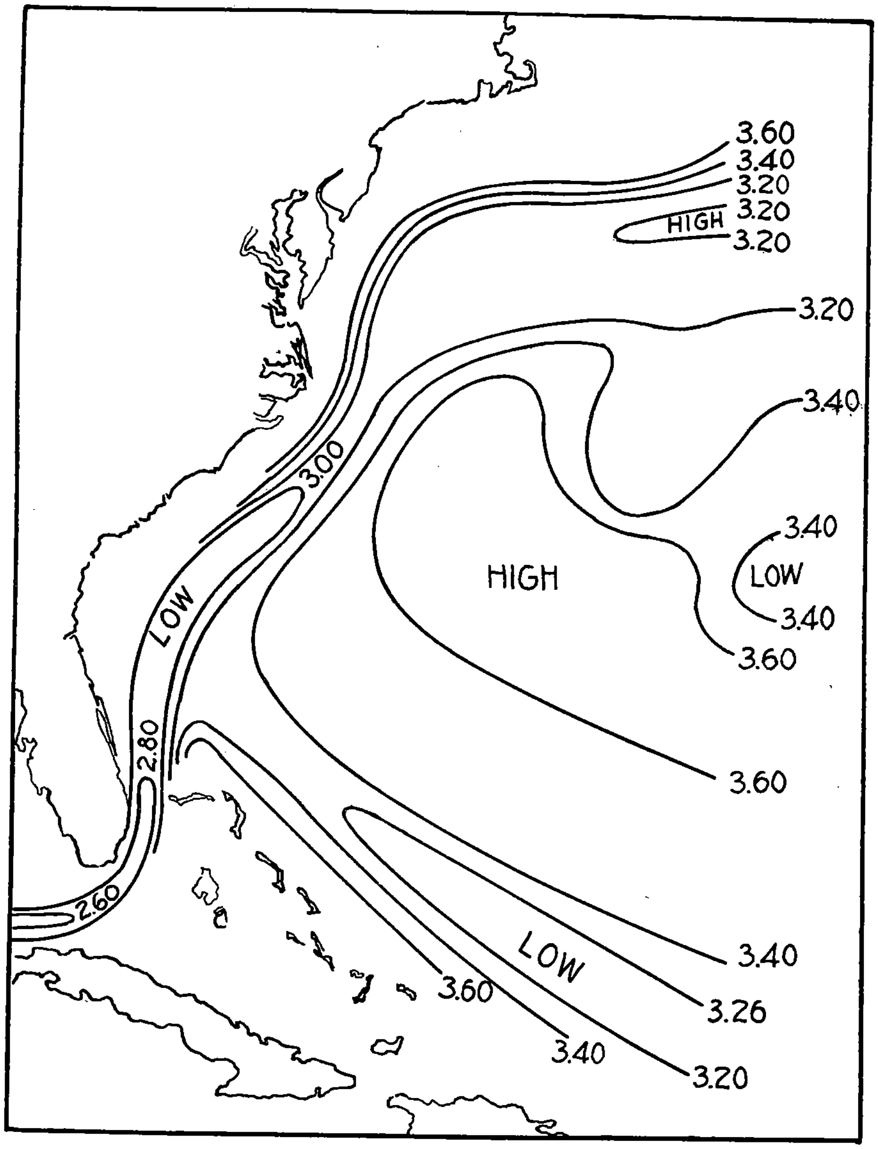

CHART II.—Preliminary chart of the horizontal distribution of oxygen minimum values.

sides of the current axis and thus the occurrence of an absolute oxygen minimum in its center appears quite unexplainable on the basis of oxidation *in situ*.

A preliminary chart of the horizontal distribution of oxygen-minimum values west of the longitude of Bermuda has been prepared and is here reproduced (Chart II). Due to the limited thickness of the minimum layer and the inadequate vertical spacing of the levels from which data are available, the details of the chart should not be given too much weight. The correct procedure would be to prepare oxygen-salinity curves for the individual stations and with their aid to estimate the correct minimum values. Nevertheless the existing data are sufficient to demonstrate the general changes in oxygen values across the curving surface of lowest readings. The chart clearly shows that the lowest readings emanate from the Straits of Florida, while a secondary tongue with somewhat higher readings is seen to be carried northwestward by the Antilles Current. The stations that were chosen above for drawing the characteristic Sargasso Sea oxygen-salinity curve lie in the tip of this secondary tongue, and we see from the chart that these stations are typical of the water absorbed by the Gulf Stream on its right side between the Bahamas and, say, the Onslow Bay section.

Beyond the Onslow Bay section the minimum oxygen on the right is above 3.60. This may account for the character of the oxygen-salinity curve for station 1227 (Fig. 22) in the right side of the Stream off Chesapeake Bay, which lies above the characteristic Sargasso curve "S". The oxygen-salinity curve for station 1226 lies below the curve "S"; this, as well as other minor discrepancies in the oxygen-salinity curves, may be explained by the presence of the minor eddies which effect the lateral stresses and lateral mixing. In particular, the water at 700 m. at station 1227 may have been brought by an eddy originating further to the right, while the corresponding water at station 1226 may have been brought by an eddy originating nearer the center of the stream. The large tongue giving values higher than 3.60 presumably has been carried in by the anticyclonic movement from the northeastern part of the Sargasso Sea.

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