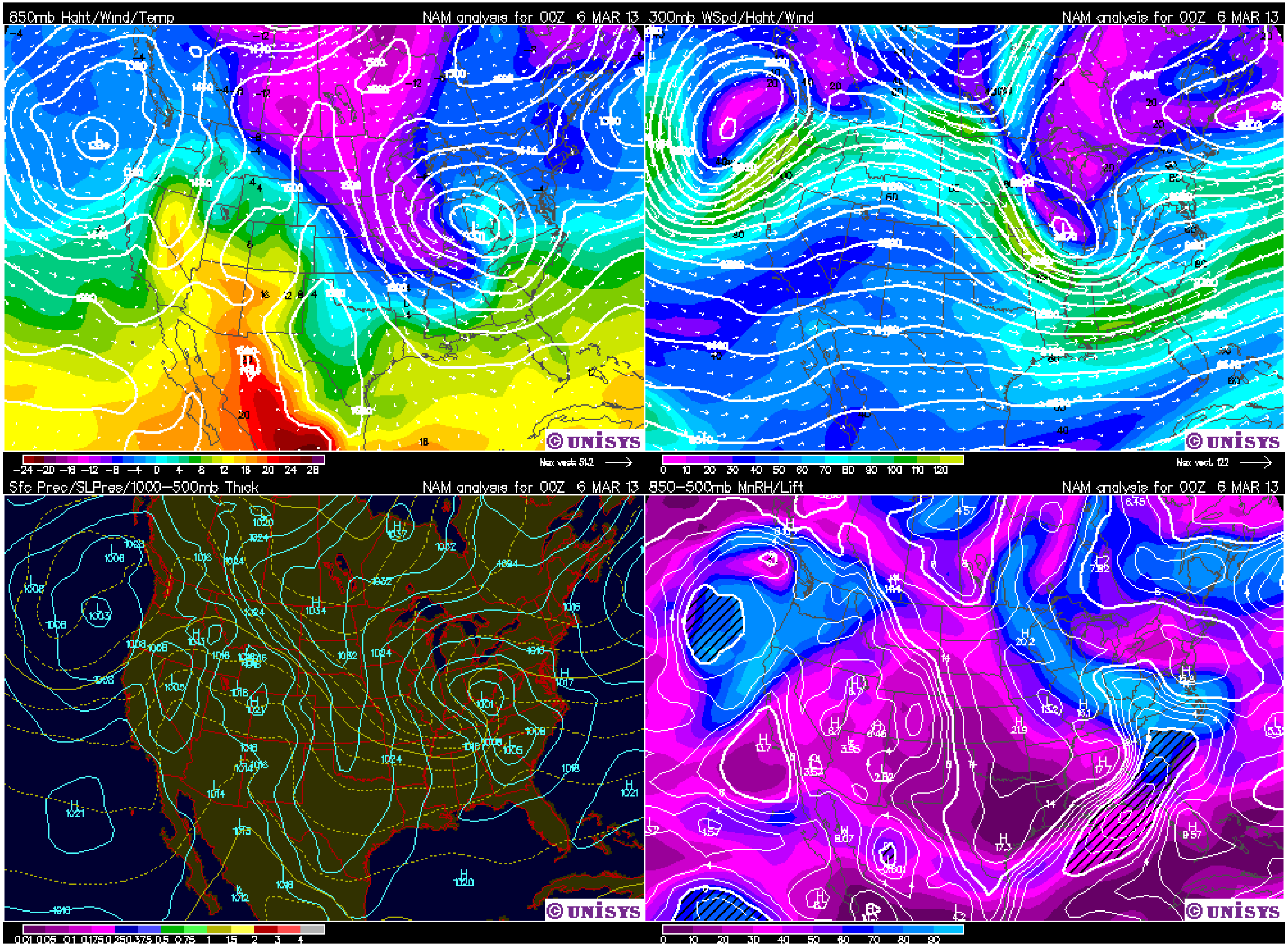
QG Analysis: Vertical Motion



QG Analysis

QG Theory

- Basic Idea
- Approximations and Validity
- QG Equations / Reference

QG Analysis

- Basic Idea
- Estimating Vertical Motion
    - QG Omega Equation: Basic Form
    - QG Omega Equation: Relation to Jet Streaks
    - QG Omega Equation: Q-vector Form
- Estimating System Evolution
    - QG Height Tendency Equation
- Diabatic and Orographic Processes
- Evolution of Low-level Cyclones
- Evolution of Upper-level Troughs

QG Analysis: Basic Idea

Forecast Needs:

- The public desires information regarding temperature, humidity, precipitation, and wind speed and direction up to 7 days in advance across the entire country
- Such information is largely a function of the **evolving synoptic weather patterns** (i.e., surface pressure systems, fronts, and jet streams)

Forecast Method:

Kinematic Approach: **Analyze current observations of wind, temperature, and moisture fields**
Assume clouds and precipitation occur when there is upward motion and an adequate supply of moisture
QG theory

QG Analysis:

- **Vertical Motion:** Diagnose synoptic-scale vertical motion from the observed distributions of differential geostrophic vorticity advection and temperature advection

- System Evolution: Predict changes in the local geopotential height patterns from the observed distributions of geostrophic vorticity advection and differential temperature advection

QG Analysis: Basic Idea

Estimating vertical motion in the atmosphere:

Our Challenge:

- We do not observe vertical motion
- Vertical motions influence clouds and precipitation
- Actual vertical motions are often several orders of magnitude smaller than their collocated horizontal air motions [w ~ 0.01 → 10 m/s]
[u,v ~ 10 → 100 m/s]
- Synoptic-scale vertical motions must be estimated from widely-spaced observations (i.e., the rawinsonde network) every 12-hours

Methods:

• Kinematic Method	Integrate the Continuity Equation Very sensitive to small errors in winds measurements
• Adiabatic Method	From the thermodynamic equation Very sensitive to temperature tendencies (difficult to observe) Difficult to incorporate impacts of diabatic heating
▪ QG Omega Equation	Least sensitive to small observational errors Widely believed to be the best method

QG Analysis: A Closed System of Equations

Two Prognostic Equations – We Need Two Unknowns:

- In order to analyze vertical motion, we need to combine our two primary *prognostic* equations – for $\boldsymbol{\zeta_g}$ and **T** – into a single equation for $\boldsymbol{\omega}$

$$\frac{\partial \zeta_g}{\partial t} = -V_g \bullet \nabla(\zeta_g + f) + f_0 \frac{\partial \omega}{\partial p}$$

Vorticity Equation

$$\frac{\partial T}{\partial t} = -V_g \bullet \nabla T + \omega\sigma \frac{p}{R}$$

Adiabatic Thermodynamic Equation

- These 2 equations have 3 prognostic variables ($\boldsymbol{\zeta_g}$, **T**, and $\boldsymbol{\omega}$) → we want to keep $\boldsymbol{\omega}$
- We need to convert both $\boldsymbol{\zeta_g}$ and **T** into a common prognostic variable

Common Variable: Geopotential-Height Tendency (χ):

- We define a local change (or tendency) in geopotential-height:

$$\chi = \frac{\partial \Phi}{\partial t}$$

where $\Phi \equiv g\, z$

QG Analysis: A Closed System of Equations

Expressing Vorticity in terms of Geopotential Height:

• Begin with the definition of geostrophic relative vorticity:

$$\zeta_g = \frac{\partial v_g}{\partial x} - \frac{\partial u_g}{\partial y} \qquad \text{where} \qquad u_g \equiv -\frac{1}{f_0}\frac{\partial \Phi}{\partial y} \qquad v_g \equiv \frac{1}{f_0}\frac{\partial \Phi}{\partial x}$$

• Substitute using the geostrophic wind relations, and one can easily show:

$$\zeta_g = \frac{1}{f_0}\nabla^2\Phi \qquad \text{where} \qquad \nabla^2\Phi = \nabla \bullet \nabla\ \Phi = \frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2}$$

• We can now define local changes in geostrophic vorticity in terms of geopotential height and local height tendency (on pressure surfaces)

$$\frac{\partial \zeta_g}{\partial t} = \frac{\partial}{\partial t}\left(\frac{1}{f_0}\nabla^2\Phi\right) = \frac{1}{f_0}\nabla^2\chi$$

QG Analysis: A Closed System of Equations

Expressing Temperature in terms of Geopotential Height:

- Begin with the hydrostatic relation in isobaric coordinates:

$$\frac{\partial \Phi}{\partial p} = -\frac{RT}{p}$$

- Using some algebra, one can easily show:

$$T = -\frac{p}{R}\frac{\partial \Phi}{\partial p}$$

- We can now define local changes in temperature in terms of geopotential height and local height tendency (on pressure surfaces)

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial t}\left(-\frac{p}{R}\frac{\partial \Phi}{\partial p}\right) = -\frac{p}{R}\frac{\partial \chi}{\partial p}$$

QG Analysis: A Closed System of Equations

Two Prognostic Equations – We Need Two Unknowns:

• We can now used these relationships to construct a closed system with two prognostic equations and two prognostic variables:

$$\frac{\partial \zeta_g}{\partial t} = \frac{\partial}{\partial t}\left(\frac{1}{f_0}\nabla^2\Phi\right) = \frac{1}{f_0}\nabla^2\chi$$

$$\zeta_g = \frac{1}{f_0}\nabla^2\Phi$$

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial t}\left(-\frac{p}{R}\frac{\partial \Phi}{\partial p}\right) = -\frac{p}{R}\frac{\partial \chi}{\partial p}$$

$$T = -\frac{p}{R}\frac{\partial \Phi}{\partial p}$$

$$\frac{\partial \zeta_g}{\partial t} = -V_g \bullet \nabla(\zeta_g + f) + f_0\frac{\partial \omega}{\partial p} \quad \Rightarrow \quad \frac{1}{f_o}\nabla^2\chi = -V_g \bullet \nabla\left[\frac{1}{f_o}\nabla^2\Phi + f\right] + f_0\frac{\partial \omega}{\partial p}$$

$$\frac{\partial T}{\partial t} = -V_g \bullet \nabla T + \omega\sigma\frac{p}{R} \quad \Rightarrow \quad -\frac{p}{R}\frac{\partial \chi}{\partial p} = -V_g \bullet \nabla\left[-\frac{p}{R}\frac{\partial \Phi}{\partial p}\right] + \omega\sigma\frac{p}{R}$$

Note: **These two equations will used to obtain the QG omega equation and, eventually, the QG height-tendency equation**

QG Analysis: Vertical Motion

The QG Omega Equation:

- We can also derive a ***single*** ***diagnostic*** equation for **ω** by combining our modified vorticity and thermodynamic equations (the height-tendency versions):

$$\frac{1}{f_o}\nabla^2\chi = -V_g \bullet \nabla\left[\frac{1}{f_o}\nabla^2\Phi + f\right] + f_0\frac{\partial\omega}{\partial p}$$

$$-\frac{p}{R}\frac{\partial\chi}{\partial p} = -V_g \bullet \nabla\left[-\frac{p}{R}\frac{\partial\Phi}{\partial p}\right] + \omega\sigma\frac{p}{R}$$

- To do this, we need to eliminate the height tendency (**χ**) from both equations

Step 1: Apply the operator $-\frac{f_0}{\sigma}\frac{\partial}{\partial p}$ to the vorticity equation

Step 2: Apply the operator $\frac{R}{p\sigma}\nabla^2$ to the thermodynamic equation

Step 3: Subtract the result of Step 1 from the result of Step 2

After a lot of math, we get the resulting diagnostic equation……

QG Analysis: Vertical Motion

The QG Omega Equation:

$$\left(\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega = \frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[V_g \bullet \nabla \left(\frac{1}{f_o}\nabla^2\Phi + f\right)\right] + \frac{R}{\sigma p}\nabla^2\left[V_g \bullet \nabla\left(-\frac{p}{R}\frac{\partial \Phi}{\partial p}\right)\right]$$

- This is (2.29) in the Lackmann text
- This form of the equation is not very intuitive since we transformed geostrophic vorticity and temperature into terms of geopotential height.
- To make this equation more intuitive, let’s transform them back…

$$\zeta_g = \frac{1}{f_0}\nabla^2\Phi$$

$$T = -\frac{p}{R}\frac{\partial \Phi}{\partial p}$$

$$\left(\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega = \frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[V_g \bullet \nabla \left(\zeta_g + f\right)\right] + \frac{R}{\sigma p}\nabla^2\left(V_g \bullet \nabla T\right)$$

QG Analysis: Vertical Motion

The BASIC QG Omega Equation:

$$\underbrace{\left(\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega}_{\text{Term A}} = \underbrace{\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[\vec{V}_g \bullet \nabla\left(\zeta_g + f\right)\right]}_{\text{Term B}} + \underbrace{\frac{R}{\sigma p}\nabla^2\left(\vec{V}_g \bullet \nabla T\right)}_{\text{Term C}}$$

- To obtain an ***actual value*** for **ω** (the ideal goal), we would need to compute the forcing terms (Terms B and C) from the three-dimensional wind and temperature fields, and then invert the operator in Term A using a numerical procedure, called "successive over-relaxation", with appropriate boundary conditions
- This is NOT a simple task (*forecasters never do this*)…..

- Rather, we can ***infer the sign and relative magnitude*** of **ω** through simple inspection of the three-dimensional absolute geostrophic vorticity and temperature fields (*forecasters do this all the time…*)
- Thus, let's examine the physical interpretation of each term….

QG Analysis: Vertical Motion

The BASIC QG Omega Equation:

$$\underbrace{\left(\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega}_{\text{Term A}} = \underbrace{\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[V_g \bullet \nabla \left(\zeta_g + f\right)\right]}_{\text{Term B}} + \underbrace{\frac{R}{\sigma p}\nabla^2\left(V_g \bullet \nabla T\right)}_{\text{Term C}}$$

Term A: Local Vertical Motion

- This term is **our goal** – a qualitative estimate of the deep–layer synoptic-scale vertical motion at a particular location

- For synoptic-scale atmospheric waves, this term is **proportional to –ω**
- Given that **ω** is negative for upward motion, conveniently, **–ω** has the same sign as the height coordinate upward motion **+w**
- Thus, if we incorporate the negative sign into our physical interpretation, we can just think of this term as “traditional” vertical motion

QG Analysis: Vertical Motion

The BASIC QG Omega Equation:

$$\underbrace{\left(\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega}_{\text{Term A}} = \underbrace{\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[V_g \bullet \nabla\left(\zeta_g + f\right)\right]}_{\text{Term B}} + \underbrace{\frac{R}{\sigma p}\nabla^2\left(V_g \bullet \nabla T\right)}_{\text{Term C}}$$

Term B: Vertical Derivative of Absolute Geostrophic Vorticity Advection
(*Differential Vorticity Advection*)

Single Pressure Level:

- Positive vorticity advection (**PVA**) causes local vorticity increases

PVA $\rightarrow$ $\frac{\partial \zeta_g}{\partial t} > 0$

- From our relationship between ζ_g and χ, we know that PVA is equivalent to:

$\frac{\partial \zeta_g}{\partial t} = \frac{1}{f_0}\nabla_p^2 \chi$ therefore: **PVA** $\rightarrow$ $\nabla_p^2 \chi > 0$ or, since: $\nabla^2 \chi \propto -\chi$ **PVA** $\rightarrow$ $\chi < 0$

- Thus, we know that **PVA** at a single level leads to **height falls**
- Using similar logic, **NVA** at a single level leads to **height rises**

QG Analysis: Vertical Motion

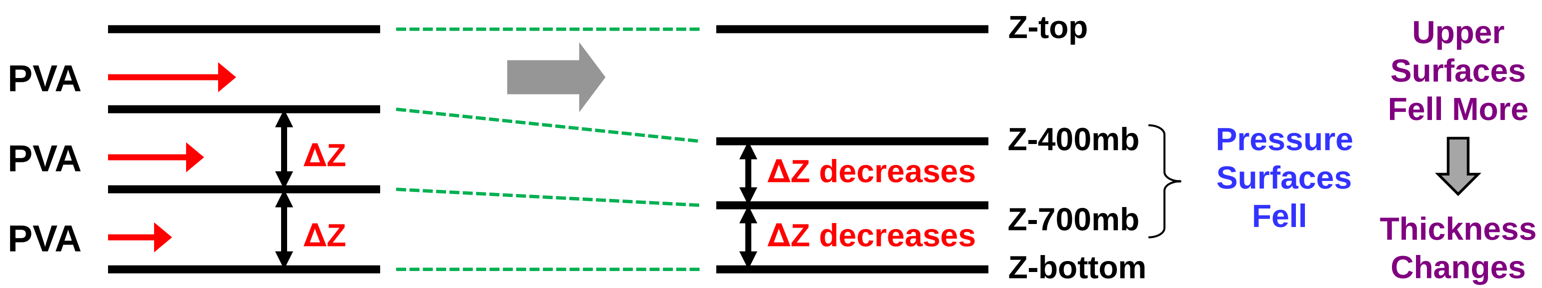
The BASIC QG Omega Equation:

$$\underbrace{\left(\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega}_{\text{Term A}} = \underbrace{\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[\vec{V}_g \bullet \nabla\left(\zeta_g + f\right)\right]}_{\text{Term B}} + \underbrace{\frac{R}{\sigma p}\nabla^2\left(\vec{V}_g \bullet \nabla T\right)}_{\text{Term C}}$$

Term B: Vertical Derivative of Absolute Geostrophic Vorticity Advection
(*Differential Vorticity Advection*)

Multiple Pressure Levels

- Consider a three-layer atmosphere where **PVA** is strongest in the upper layer:



WAIT! Hydrostatic balance (via the hypsometric equation) requires ALL changes in thickness (ΔZ) to be accompanied by temperature changes.

BUT these thickness changes were ***NOT*** a result of temperature changes…

QG Analysis: Vertical Motion

The BASIC QG Omega Equation:

$$\underbrace{\left(\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega}_{\text{Term A}} = \underbrace{\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[\vec{V}_g \bullet \nabla \left(\zeta_g + f\right)\right]}_{\text{Term B}} + \underbrace{\frac{R}{\sigma p}\nabla^2\left(\vec{V}_g \bullet \nabla T\right)}_{\text{Term C}}$$

Term B: Vertical Derivative of Absolute Geostrophic Vorticity Advection (*Differential Vorticity Advection*)

- In order to maintain ***hydrostatic balance***, any thickness decreases must be accompanied by a temperature decrease or cooling
- Recall our ***adiabatic*** assumption

↑ **Rising Motions** ⇒ **Adiabatic Cooling**

↓ **Sinking Motions** ⇒ **Adiabatic Warming**

- Therefore, in the absence of temperature advection and diabatic processes:
  - An **increase in PVA with height** will induce **rising motion**

QG Analysis: Vertical Motion

The BASIC QG Omega Equation:

$$\underbrace{\left(\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega}_{\text{Term A}} = \underbrace{\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[V_g \bullet \nabla\left(\zeta_g + f\right)\right]}_{\text{Term B}} + \underbrace{\frac{R}{\sigma p}\nabla^2\left(V_g \bullet \nabla T\right)}_{\text{Term C}}$$

Term B: Vertical Derivative of Absolute Geostrophic Vorticity Advection
(*Differential Vorticity Advection*)

Possible **rising motion** scenarios: Strong **PVA** in upper levels
Weak **PVA** in lower levels

PVA in upper levels
No vorticity advection in lower levels

PVA in upper levels
NVA in lower levels

Weak **NVA** in upper levels
Strong **NVA** in lower levels

QG Analysis: Vertical Motion

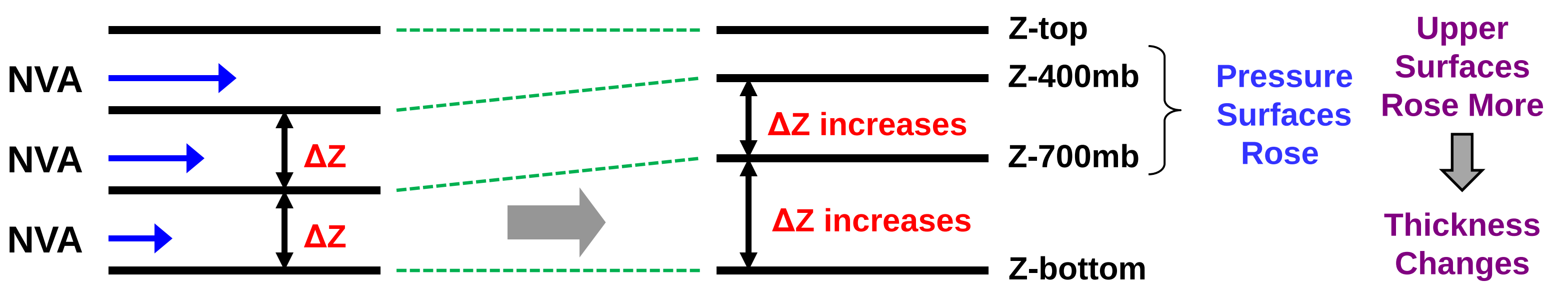
The BASIC QG Omega Equation:

$$\underbrace{\left(\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega}_{\text{Term A}} = \underbrace{\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[\vec{V}_g \bullet \nabla\left(\zeta_g + f\right)\right]}_{\text{Term B}} + \underbrace{\frac{R}{\sigma p}\nabla^2\left(\vec{V}_g \bullet \nabla T\right)}_{\text{Term C}}$$

Term B: Vertical Derivative of Absolute Geostrophic Vorticity Advection
(*Differential Vorticity Advection*)

Multiple Pressure Levels

- Consider a three-layer atmosphere where **NVA** is strongest in the upper layer:



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BUT these thickness changes were ***NOT*** a result of temperature changes…

QG Analysis: Vertical Motion

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Term B: Vertical Derivative of Absolute Geostrophic Vorticity Advection (*Differential Vorticity Advection*)

- In order to maintain ***hydrostatic balance***, any thickness increases must be accompanied by a temperature increase or warming
- Recall our ***adiabatic*** assumption

↑ **Rising Motions** ⇒ **Adiabatic Cooling**

↓ **Sinking Motions** ⇒ **Adiabatic Warming**

- Therefore, in the absence of temperature advection and diabatic processes:
  - An **increase in NVA with height** will induce **sinking motion**

QG Analysis: Vertical Motion

The BASIC QG Omega Equation:

$$\underbrace{\left(\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega}_{\text{Term A}} = \underbrace{\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[V_g \bullet \nabla\left(\zeta_g + f\right)\right]}_{\text{Term B}} + \underbrace{\frac{R}{\sigma p}\nabla^2\left(V_g \bullet \nabla T\right)}_{\text{Term C}}$$

Term B: Vertical Derivative of Absolute Geostrophic Vorticity Advection
(*Differential Vorticity Advection*)

Possible **rising motion** scenarios: Strong **NVA** in upper levels
Weak **NVA** in lower levels

NVA in upper levels
No vorticity advection in lower levels

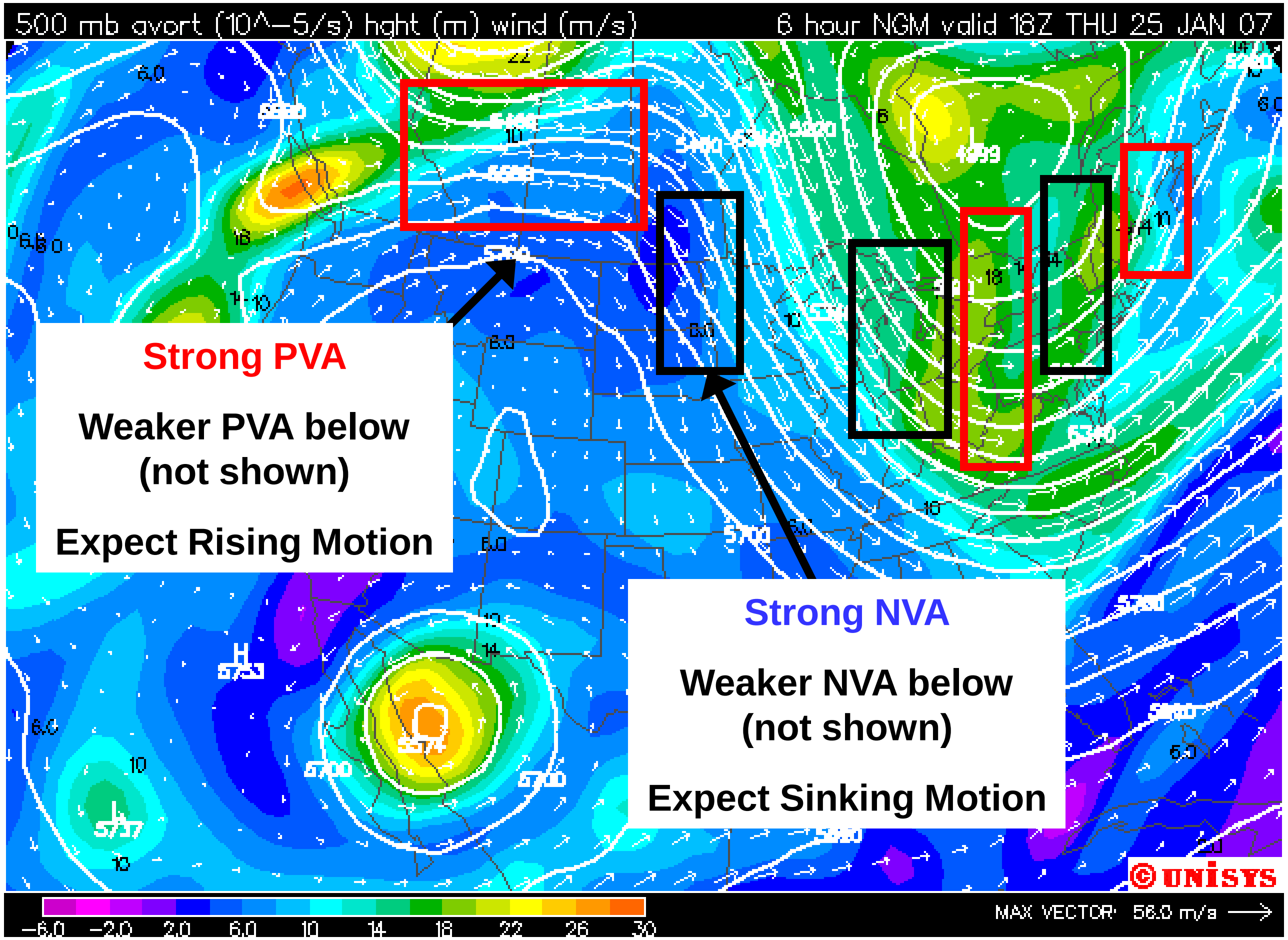
NVA in upper levels
PVA in lower levels

Weak **PVA** in upper levels
Strong **PVA** in lower levels

QG Analysis: Vertical Motion

The BASIC QG Omega Equation:

Term B: Vertical Derivative of Absolute Geostrophic Vorticity Advection
(*Differential Vorticity Advection*)

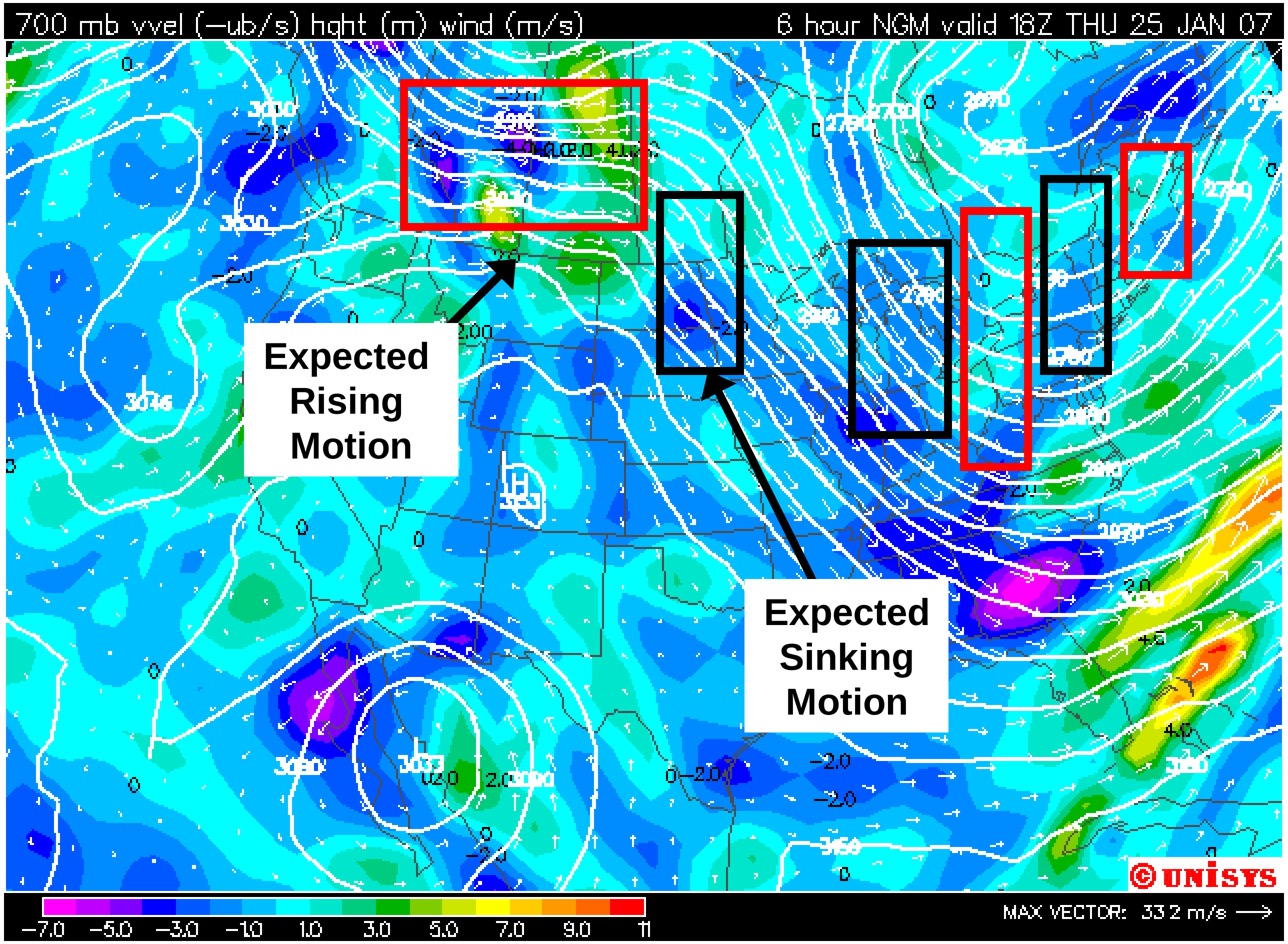


Full-Physics Model Analysis

QG Analysis: Vertical Motion

The BASIC QG Omega Equation:

Term B: Vertical Derivative of Absolute Geostrophic Vorticity Advection
(*Differential Vorticity Advection*)



Generally consistent with expectations!

QG Analysis: Vertical Motion

The BASIC QG Omega Equation:

Term B: Vertical Derivative of Absolute Geostrophic Vorticity Advection
(*Differential Vorticity Advection*)

Generally Consistent…BUT Noisy → Why?

- Only evaluated one level (500mb) → should evaluate multiple levels
- Used full wind and vorticity fields → should use geostrophic wind and vorticity
- Mesoscale-convective processes → QG focuses on only synoptic-scale (small R_o)
- Condensation / Evaporation → neglected diabatic processes
- Complex terrain → neglected orographic effects
- Did not consider temperature (thermal) advection (**Term C**)!!!

- Yet, despite all these caveats, the analyzed vertical motion pattern is qualitatively consistent with expectations from the QG omega equation!!!

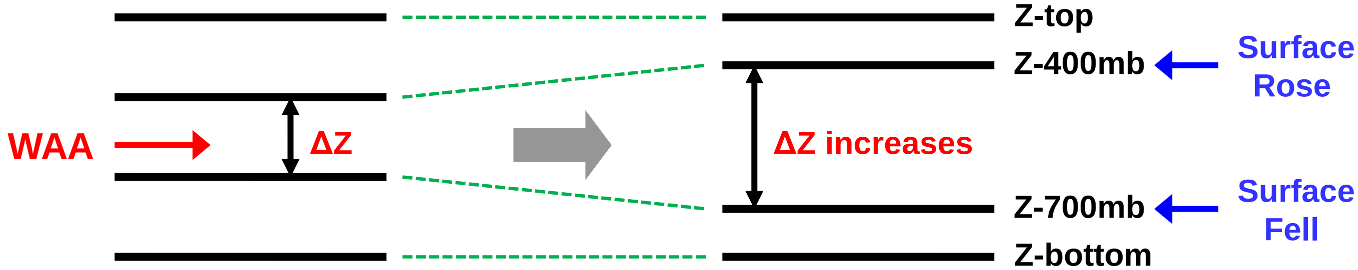
QG Analysis: Vertical Motion

The BASIC QG Omega Equation:

$$\underbrace{\left(\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega}_{\text{Term A}} = \underbrace{\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[\vec{V}_g \bullet \nabla\left(\zeta_g + f\right)\right]}_{\text{Term B}} + \underbrace{\frac{R}{\sigma p}\nabla^2\left(\vec{V}_g \bullet \nabla T\right)}_{\text{Term C}}$$

Term C: Geostrophic Temperature Advection (*Thermal Advection*)

- Warm air advection (**WAA**) leads to local temperature / thickness increases
- Consider the three-layer model, with **WAA** strongest in the middle layer



WAIT! Local geopotential height rises (falls) produce changes in the local height gradients → changing the local geostrophic wind and vorticity

BUT these thickness changes were ***NOT*** a result of geostrophic vorticity changes…

QG Analysis: Vertical Motion

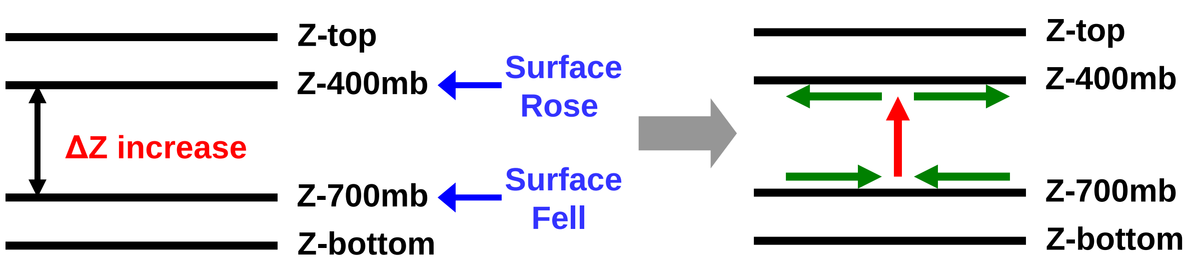
The BASIC QG Omega Equation:

$$\left(\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega = \frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[V_g \bullet \nabla \left(\zeta_g + f\right)\right] + \frac{R}{\sigma p}\nabla^2\left(V_g \bullet \nabla T\right)$$

Term A | Term B | Term C

Term C: Geostrophic Temperature Advection (*Thermal Advection*)

- In order to maintain ***geostrophic flow***, any thickness changes must be accompanied by *ageostrophic* divergence (convergence) in regions of height rises (falls), which via mass continuity requires a vertical motion through the layer



QG Mass Continuity

$$\frac{\partial u_{ag}}{\partial x} + \frac{\partial v_{ag}}{\partial y} = -\frac{\partial \omega}{\partial p}$$

- Therefore, in the absence of geostrophic vorticity advection and diabatic processes:
  - **WAA** will induce **rising motion**

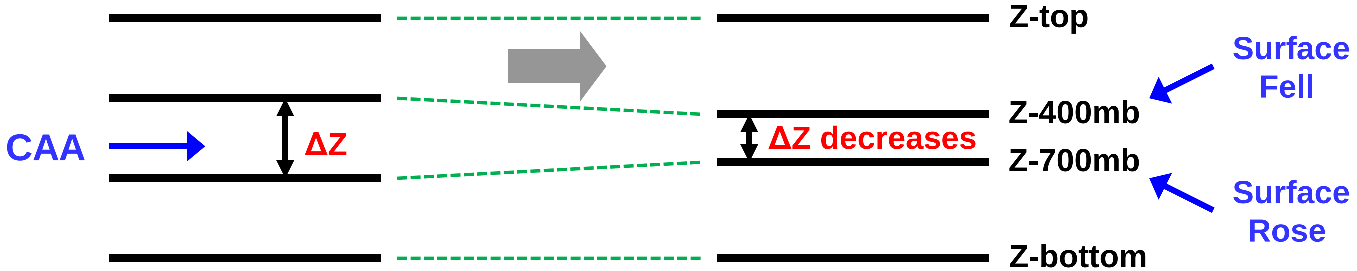
QG Analysis: Vertical Motion

The BASIC QG Omega Equation:

$$\underbrace{\left(\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega}_{\text{Term A}} = \underbrace{\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[V_g \bullet \nabla\left(\zeta_g + f\right)\right]}_{\text{Term B}} + \underbrace{\frac{R}{\sigma p}\nabla^2\left(V_g \bullet \nabla T\right)}_{\text{Term C}}$$

Term C: Geostrophic Temperature Advection (*Thermal Advection*)

- Cold air advection (**CAA**) leads to local temperature / thickness decreases
- Consider the three-layer model, with **CAA** strongest in the middle layer



WAIT! Local geopotential height rises (falls) produce changes in the local height gradients → changing the local geostrophic wind and vorticity

BUT these thickness changes were ***NOT*** a result of geostrophic vorticity changes…

QG Analysis: Vertical Motion

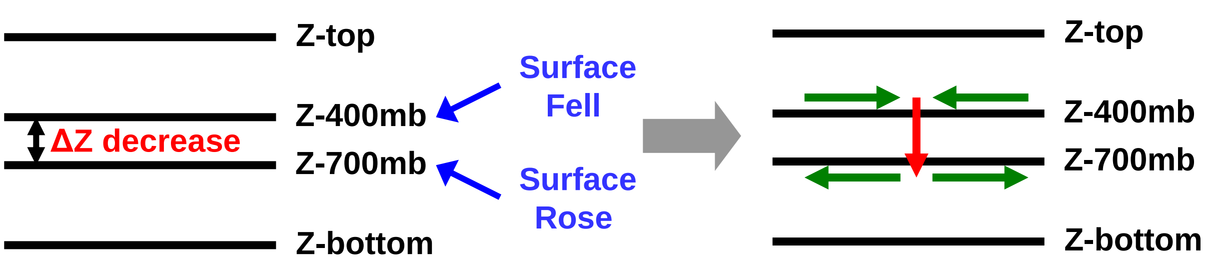
The BASIC QG Omega Equation:

$$\left(\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega = \frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[\vec{V}_g \bullet \nabla\left(\zeta_g + f\right)\right] + \frac{R}{\sigma p}\nabla^2\left(\vec{V}_g \bullet \nabla T\right)$$

Term A | Term B | Term C

Term C: Geostrophic Temperature Advection (*Thermal Advection*)

- In order to maintain ***geostrophic flow***, any thickness changes must be accompanied by *ageostrophic* divergence (convergence) in regions of height rises (falls), which via mass continuity requires a vertical motion through the layer



QG Mass Continuity

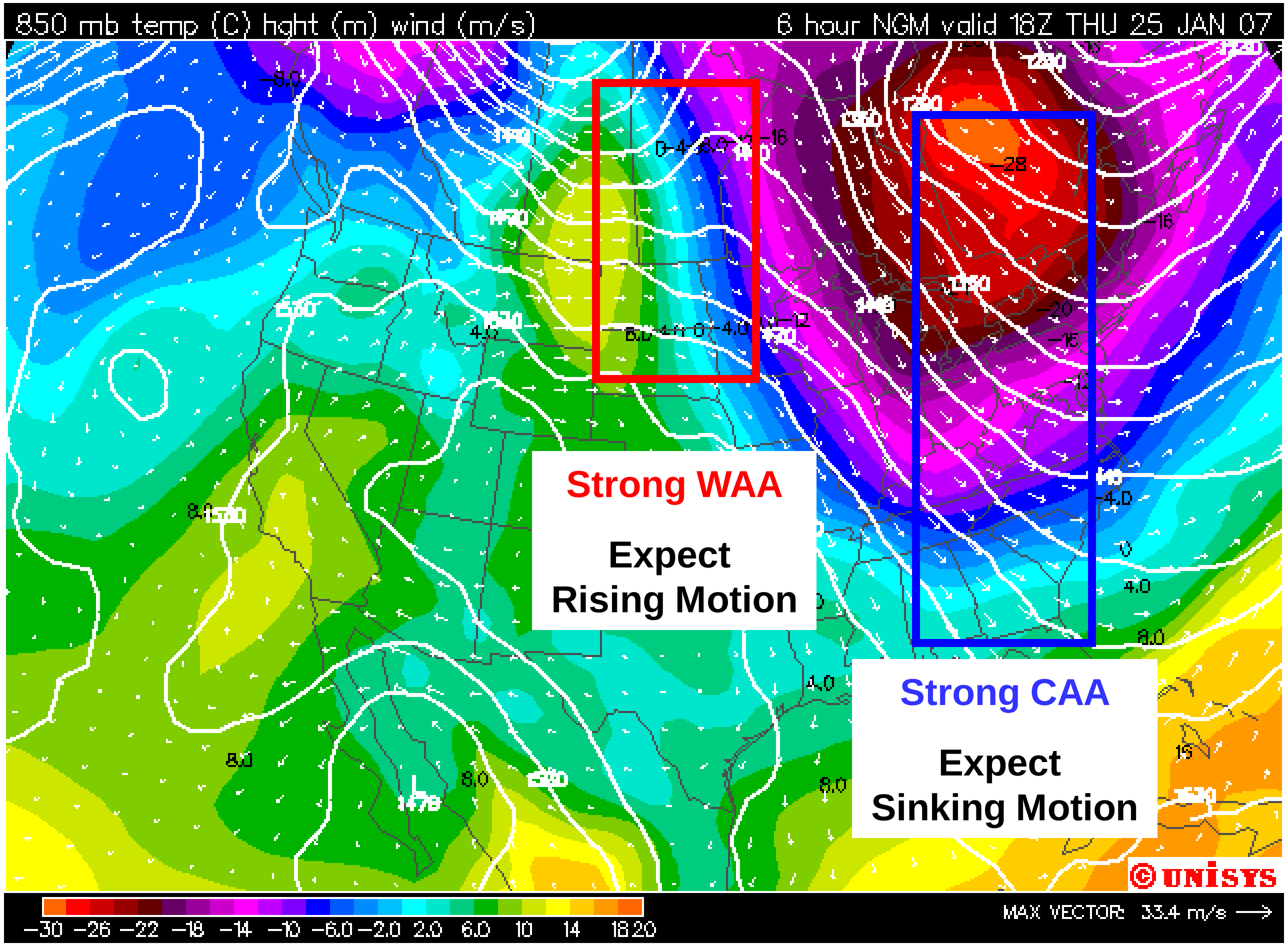
$$\frac{\partial u_{ag}}{\partial x} + \frac{\partial v_{ag}}{\partial y} = -\frac{\partial \omega}{\partial p}$$

- Therefore, in the absence of geostrophic vorticity advection and diabatic processes:
  - **CAA** will induce **sinking motion**

QG Analysis: Vertical Motion

The BASIC QG Omega Equation:

Term C: Geostrophic Temperature Advection (*Thermal Advection*)

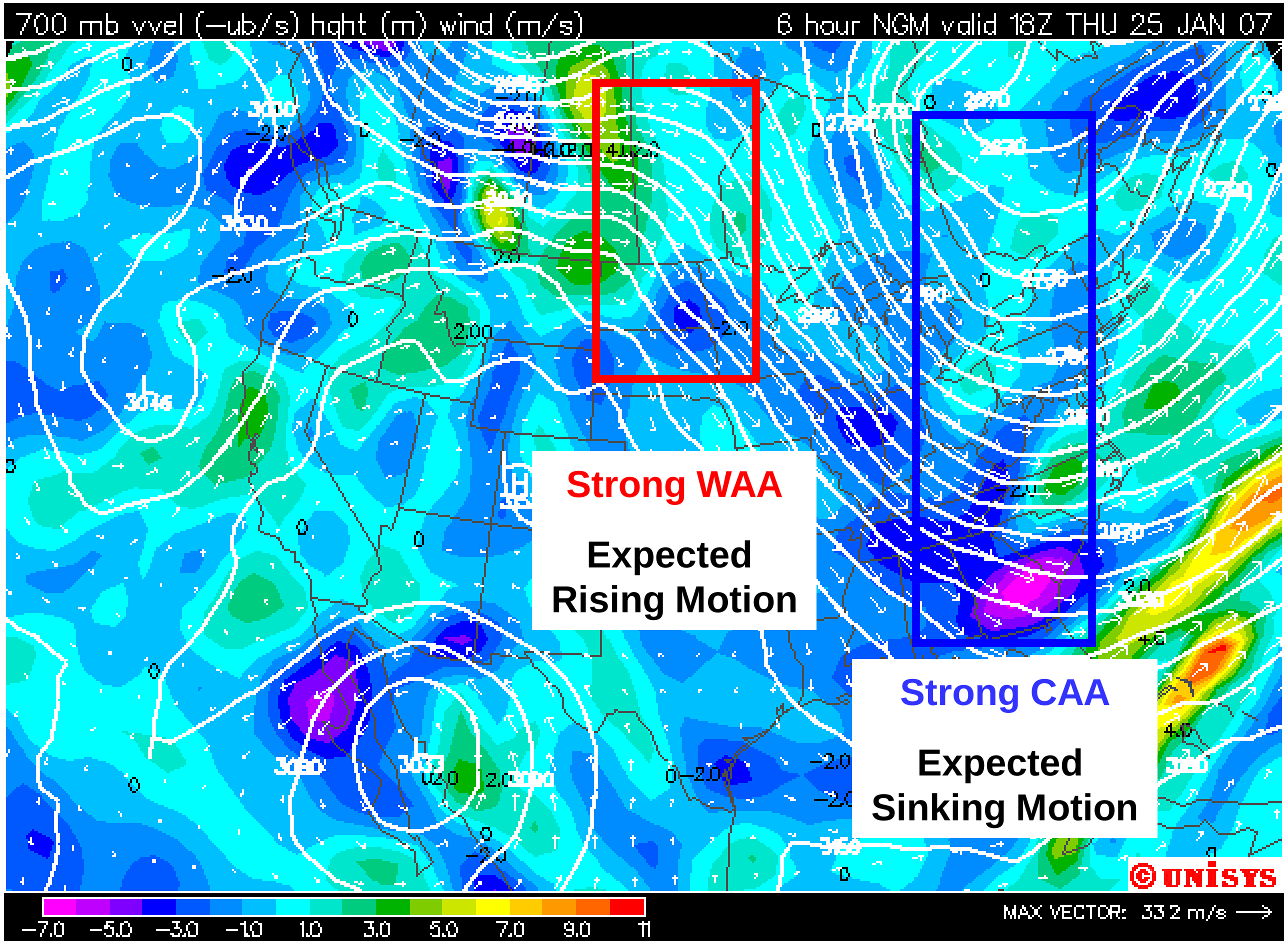


Full-Physics Model Analysis

QG Analysis: Vertical Motion

The BASIC QG Omega Equation:

Term C: Geostrophic Temperature Advection (*Thermal Advection*)



Somewhat consistent with expectations…

QG Analysis: Vertical Motion

The BASIC QG Omega Equation:

Term C: Geostrophic Temperature Advection (*Thermal Advection*)

Somewhat Consistent…BUT very noisy → Why?

- Used full wind field → should use geostrophic wind
- Only evaluated one level (850mb) → should evaluate multiple levels
- Mesoscale-convective processes → QG focuses on only synoptic-scale (small R_o)
- Condensation / Evaporation→ neglected diabatic processes
- Complex terrain → neglected orographic effects
- Did not consider differential vorticity advection (**Term B**)!!!

- Yet, despite all these caveats, the analyzed vertical motion pattern is still somewhat consistent with expectations from the QG omega equation!!!

QG Analysis: Vertical Motion

The BASIC QG Omega Equation:

$$\underbrace{\left(\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega}_{\text{Term A}} = \underbrace{\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[V_g \bullet \nabla\left(\zeta_g + f\right)\right]}_{\text{Term B}} + \underbrace{\frac{R}{\sigma p}\nabla^2\left(V_g \bullet \nabla T\right)}_{\text{Term C}}$$

Application Tips:

- Remember the **underlying assumptions**!!!

- You must consider the effects of **both** **Term B** and **Term C** at multiple levels!!!
  - If differential vorticity advection is large (small), then you should expect a correspondingly large (small) vertical motion through that layer
  - The stronger the temperature advection, the stronger the vertical motion
  - If WAA (CAA) is observed at several consecutive pressure levels, expect a deep layer of rising (sinking) motion

- **Opposing expectations** from the two terms at a given location will weaken the total vertical motion (and complicate the interpretation)!!! [more on this later]

QG Analysis: Vertical Motion

The BASIC QG Omega Equation:

$$\underbrace{\left(\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega}_{\text{Term A}} = \underbrace{\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[V_g \bullet \nabla \left(\zeta_g + f\right)\right]}_{\text{Term B}} + \underbrace{\frac{R}{\sigma p}\nabla^2\left(V_g \bullet \nabla T\right)}_{\text{Term C}}$$

Application Tips:

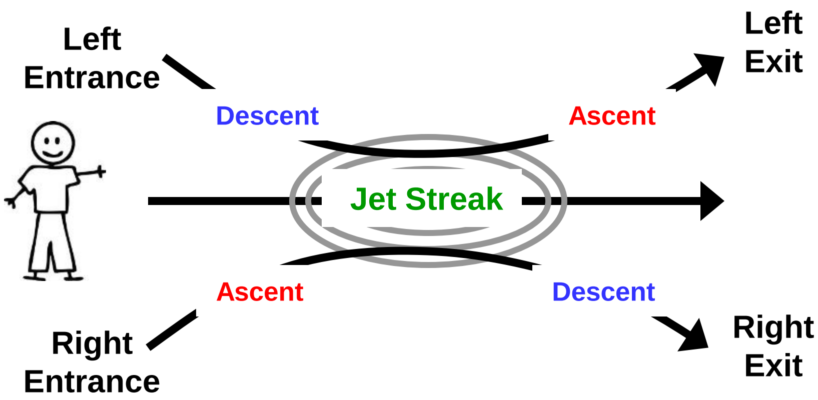
- The QG omega equation is a **diagnostic** equation:
    - The equation does ***not predict*** future vertical motion patterns
    - The forcing functions (Terms B and C) produce *instantaneous* responses

- Use of the QG omega equation in a **diagnostic** setting:
    - Diagnose the *synoptic–scale vertical motion pattern*, and assume rising motion corresponds to clouds and precipitation *when ample moisture is available*
    - Compare to the observed patterns → can infer mesoscale contributions
    - Helps distinguish between areas of persistent light precipitation (synoptic-scale) and more sporadic intense precipitation (mesoscale)

QG Analysis: Application to Jet Streaks

Review of Jet Streaks:

- Air parcels accelerate just upstream into the “entrance” region and then decelerate downstream coming out of the “exit” region (for an observer facing downstream)



- Often sub-divided into quadrants:
    - Right Entrance (or R-En)
    - Left Entrance (or L-En)
    - Right Exit (or R-Ex)
    - Left Exit (or L-Ex)

- Each quadrant has an “expected” vertical motion….**WHY?**

QG Analysis: Application to Jet Streaks

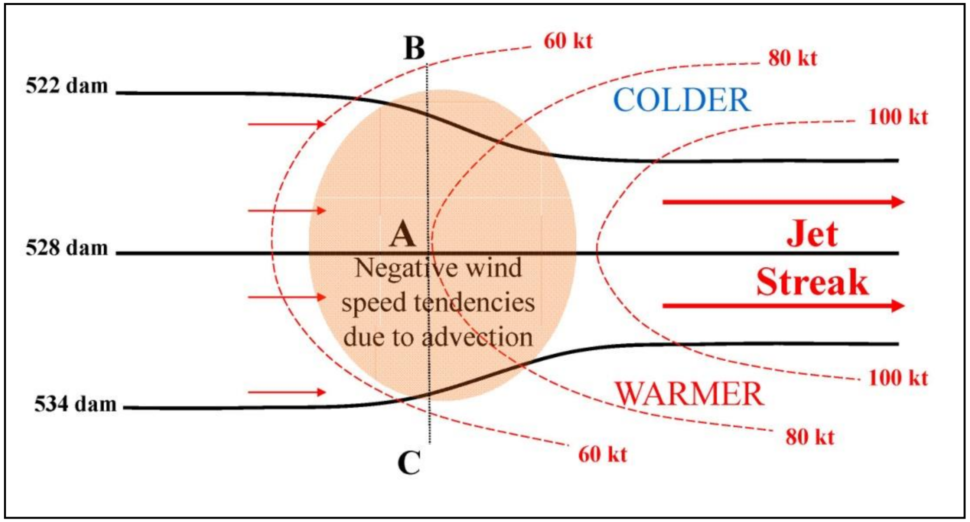
Physical Interpretation:

$$\left(\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega = \frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[\vec{V}_g \bullet \nabla\left(\zeta_g + f\right)\right] + \frac{R}{\sigma p}\nabla^2\left(\vec{V}_g \bullet \nabla T\right)$$

Term A | **Term B** | **Term C**

Basic Jet Structure / Assumptions:

- The explanation of the well-known "jet streak vertical motion pattern" lies in **Term B**
- This explanation was first advanced by Durran and Snellman (1987)
- Provided in detail by Lackmann text
- Jet streak entrance region at 500mb with structure shown to the right
- The 1000mb surface is "flat" with no height contours → *no winds*



From Lackmann (2011)

QG Analysis: Application to Jet Streaks

Physical Interpretation:

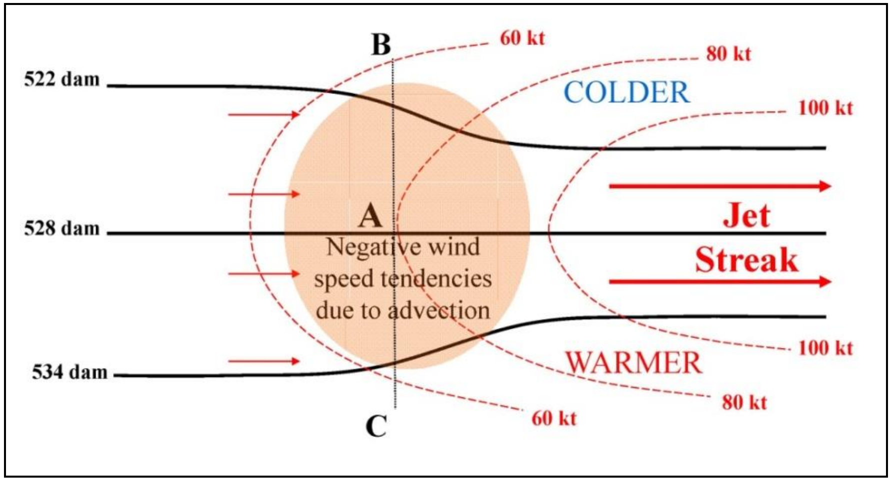
- Near point A there is a local decrease in wind speed (or a negative tendency) due to geostrophic advection
- Since the winds at 1000mb remain calm, this implies that the vertical wind shear is reduced through the entrance region

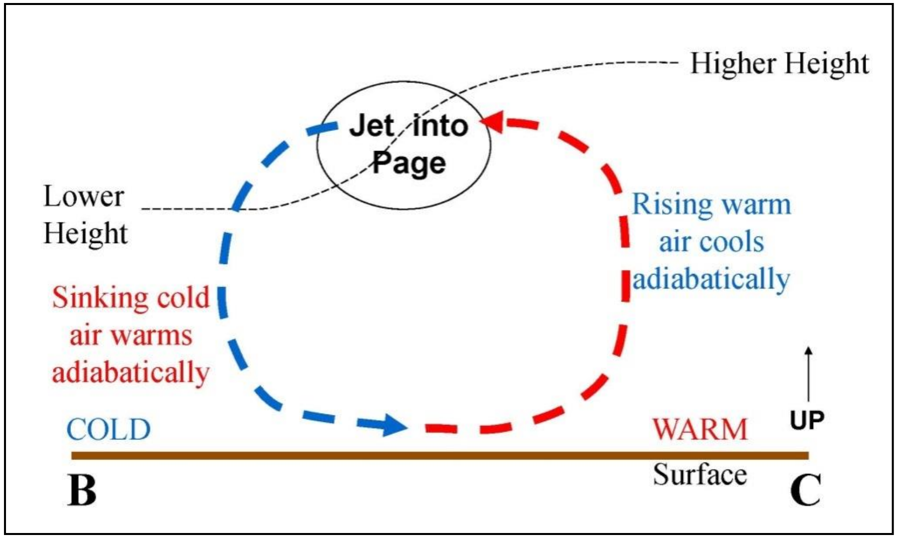
- If the wind shear decreases, *thermal wind balance is disrupted*

$$\frac{\partial u_g}{\partial p} = \frac{R}{fp}\left(\frac{\partial T}{\partial y}\right)$$

- **Something** is needed to maintain balance
  → increase in vertical shear
  → decrease in temperature gradient **

- Since *geostrophic flow disrupted balance* (!) ***ageostrophic flow*** must bring about the return to balance by weakening the thermal gradient via adiabatic vertical motions and mass continuity!





From Lackmann (2011)

QG Analysis: Application to Jet Streaks

Physical Interpretation:

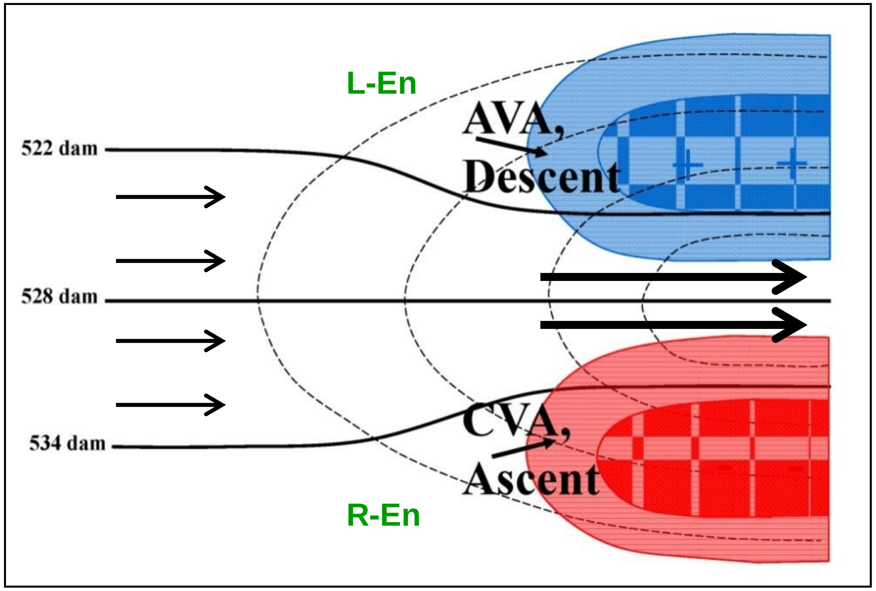
- With respect to *differential vorticity advection* (Term B), at 500mb, cyclonic vorticity (+) is located north of the jet streak, with anti-cyclonic vorticity (–) located to the south

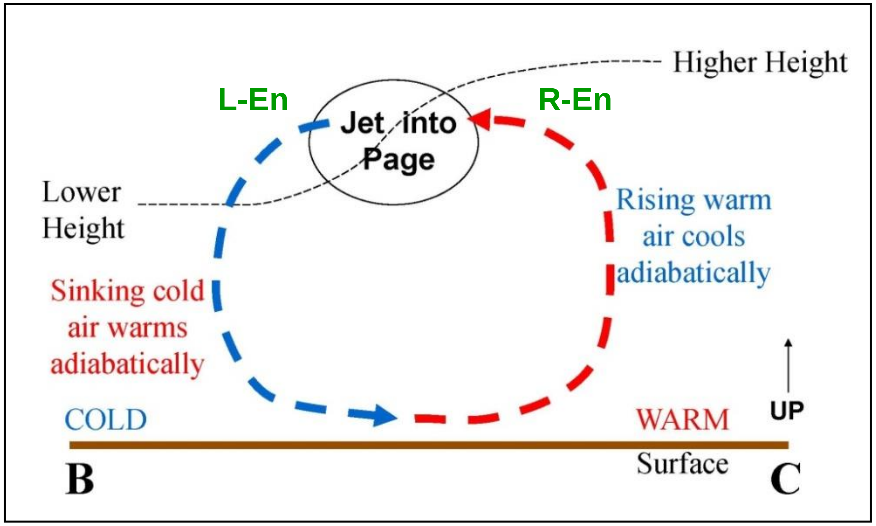
  Left Entrance region → AVA (or NVA)
  Right Entrance region → CVA (or PVA)

- With no winds at 1000mb → no vorticity advection

- Thus, evaluation of Term B implies:

  L-En → Term B < 0 → Sinking Motion
  R-En → Term B > 0 → Rising Motion



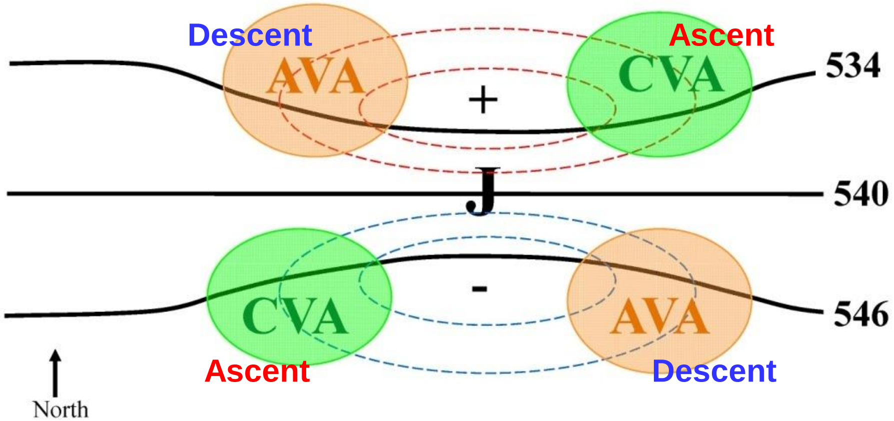


From Lackmann (2011)

QG Analysis: Application to Jet Streaks

Physical Interpretation:

• Thus, the "typical" vertical motion pattern associated with jet streaks arises from QG forcing associated with differential vorticity advection!



Important Points:

- *__The atmosphere is constantly advecting itself out of thermal wind balance__*. Even advection by the geostrophic flow can destroy balance.

- *__Ageostrophic secondary circulations__*, with vertical air motions, arise as a response and *__return the atmosphere to balance__*

QG Analysis: Q-vectors

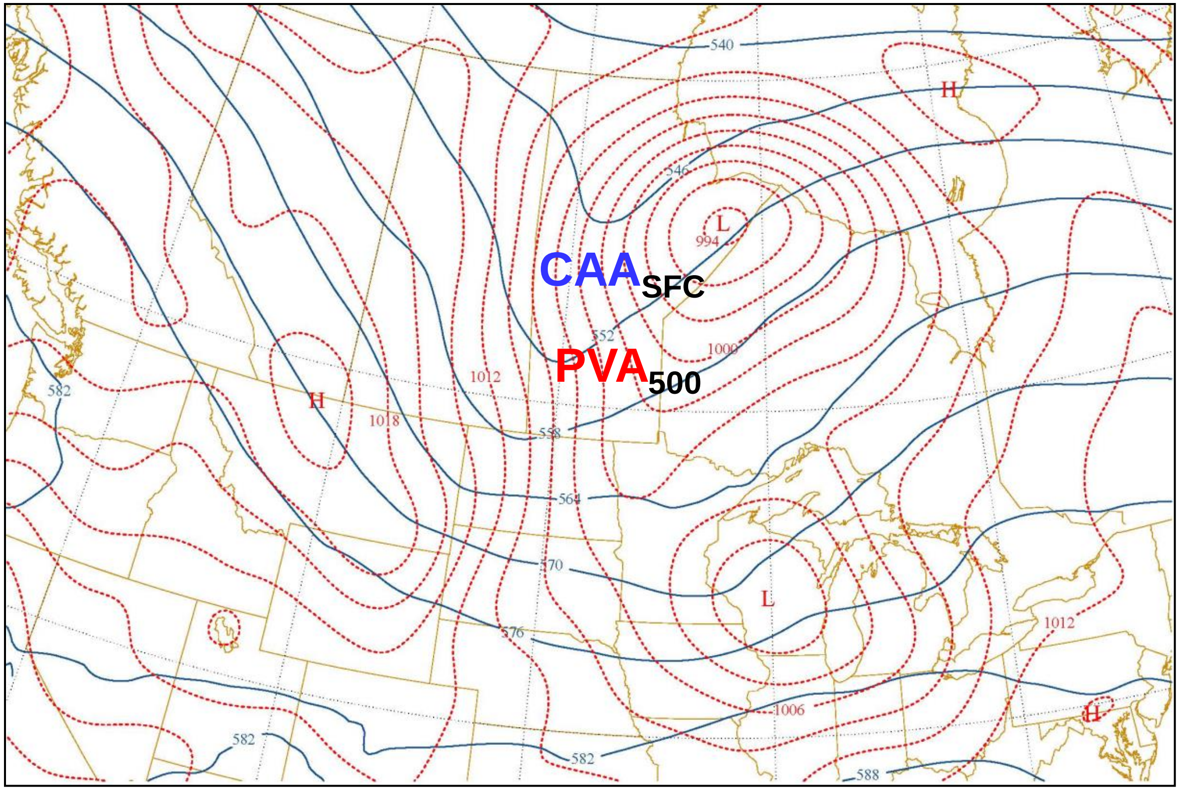
Motivation:

- Application of the ***basic*** QG omega equation involves analyzing two terms (B and C) that can (and often do) provide *opposite forcing*.

- In such cases the forecaster must estimate which forcing term is larger (or dominant)
- Dedicated forecasters find such situations and "unsatisfactory"

- The example to the right provides a case where thermal advection (Term C) and differential vorticity advection (Term B) provide opposite QG forcing

  Term B → Ascent
  Term C → Descent



- The *Q-vector form* of the QG omega equation provides a way around this issue…

QG Analysis: Q-vectors

Definition and Formulation:

- Derivation of the Q-vector form is not provided [Advanced Dynamics???]
- See *Hoskins et al. (1978)* and *Hoskins and Pedder (1980)*

$$\left(\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega = -\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[-V_g \bullet \nabla\left(\zeta_g + f\right)\right] + -\frac{R}{\sigma p}\nabla^2\left(-V_g \bullet \nabla T\right)$$

$$\left(\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega = -2\nabla \bullet \mathbf{Q}$$

Q-vector Form of the QG Omega Equation

where:

$$\mathbf{Q} = \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} = -\frac{R}{\sigma p}\begin{pmatrix} \dfrac{\partial V_g}{\partial x} \bullet \nabla\theta \\ \dfrac{\partial V_g}{\partial y} \bullet \nabla\theta \end{pmatrix} = -\frac{R}{\sigma p}\begin{pmatrix} \dfrac{\partial u_g}{\partial x}\dfrac{\partial \theta}{\partial x} + \dfrac{\partial v_g}{\partial x}\dfrac{\partial \theta}{\partial y} \\ \dfrac{\partial u_g}{\partial y}\dfrac{\partial \theta}{\partial x} + \dfrac{\partial v_g}{\partial y}\dfrac{\partial \theta}{\partial y} \end{pmatrix}$$

QG Analysis: Q-vectors

Physical Interpretation:

$$\left(\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega = -2\nabla \bullet \mathbf{Q}$$

where

$$\mathbf{Q} = \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} = -\frac{R}{\sigma p}\begin{pmatrix} \frac{\partial u_g}{\partial x}\frac{\partial \theta}{\partial x} + \frac{\partial v_g}{\partial x}\frac{\partial \theta}{\partial y} \\ \frac{\partial u_g}{\partial y}\frac{\partial \theta}{\partial x} + \frac{\partial v_g}{\partial y}\frac{\partial \theta}{\partial y} \end{pmatrix}$$

- The components $\mathbf{Q_1}$ and $\mathbf{Q_2}$ provide a measure of the horizontal wind shear across a temperature gradient in the <u>zonal and meridional directions</u>
- The two components can be combined to produce a horizontal "**Q-vector**"

- Q-vectors are oriented parallel to the ageostrophic wind vector
- Q-vectors are proportional to the magnitude of the ageostrophic wind
- Q-vectors point toward rising motion

<u>In regions where</u>:

Q-vectors converge → Ascent

Q-vectors diverge → Descent

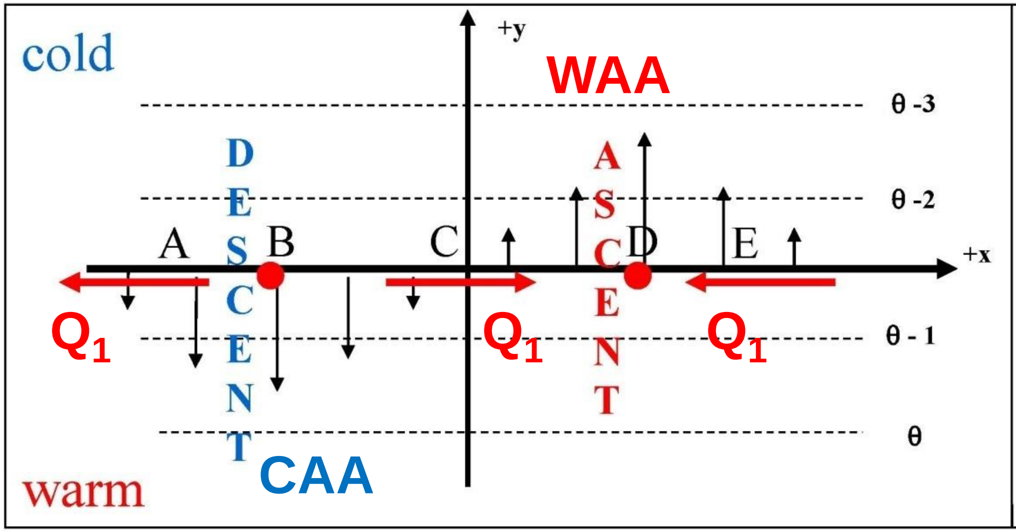
QG Analysis: Q-vectors

Physical Interpretation: **Hypothetical Case**

- Synoptic-scale low pressure system (center at C)
- Meridional flow shown by black vectors (no zonal flow)
- Warm air to the south and cold air to the north (no zonal thermal gradient)

- Regions of Q-vector forcing for vertical motion are exactly consistent with what one would expect from the basic form of the QG omega equation

WAA → Ascent
CAA → Descent



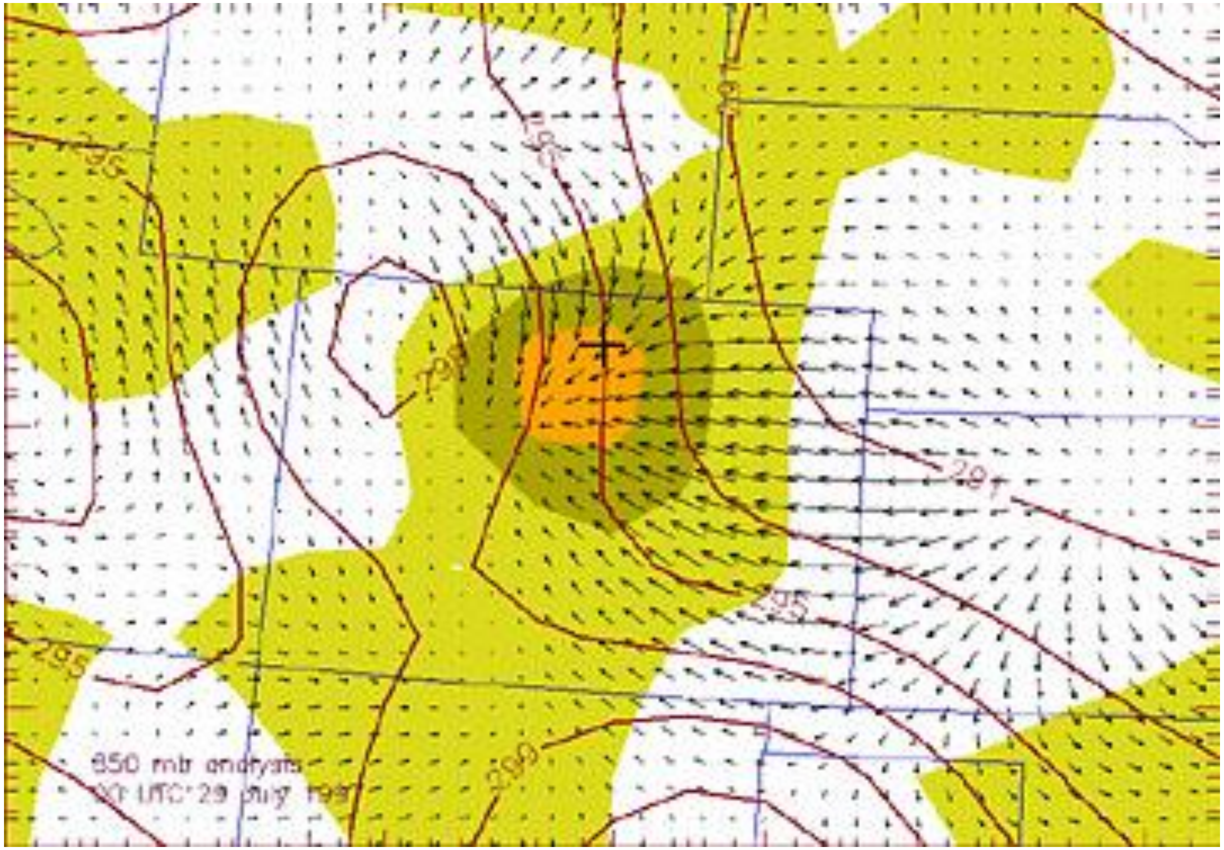
$$Q_1 \propto -\frac{\partial u_g}{\partial x}\frac{\partial \theta}{\partial x} - \frac{\partial v_g}{\partial x}\frac{\partial \theta}{\partial y}$$

$$Q_2 \propto -\frac{\partial u_g}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial v_g}{\partial y}\frac{\partial \theta}{\partial y}$$

QG Analysis: Q-vectors

Example:

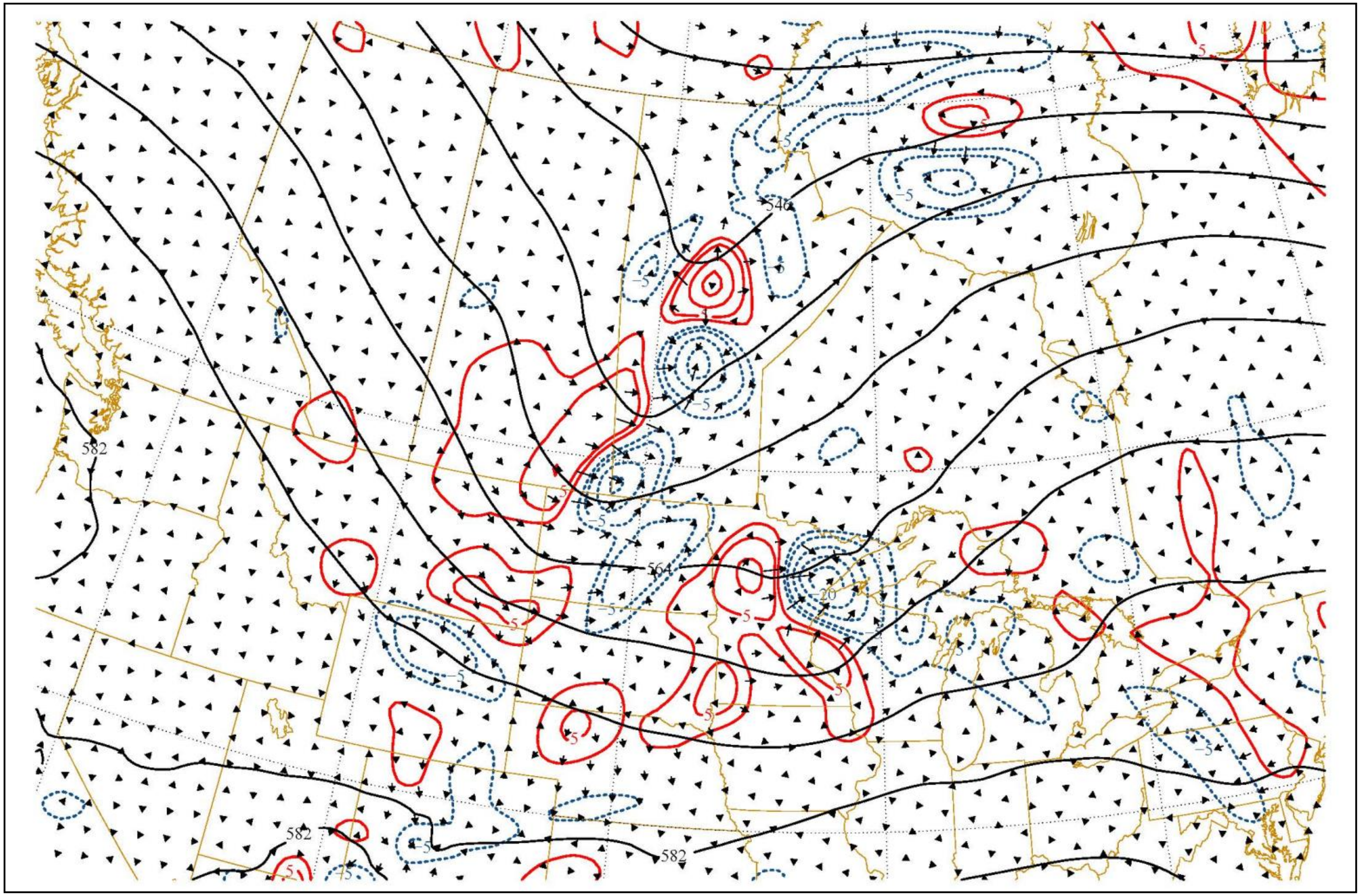
850-mb Analysis – 29 July 1997 at 00Z
Isentropes (red), Q-vectors, Vertical motion (shading, upward only)



QG Analysis: Q-vectors

Example:

500-mb GFS Forecast – 13 September 2008 at 1800Z
500-mb Heights (black), Q-vectors, Q-vector convergence (blue) and divergence (red)



QG Analysis: Q-vectors

Application Tips:

$$\left(\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega = -2\nabla \bullet \mathbf{Q}$$

where

$$\mathbf{Q} = \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} = -\frac{R}{\sigma p}\begin{pmatrix} \frac{\partial u_g}{\partial x}\frac{\partial \theta}{\partial x} + \frac{\partial v_g}{\partial x}\frac{\partial \theta}{\partial y} \\ \frac{\partial u_g}{\partial y}\frac{\partial \theta}{\partial x} + \frac{\partial v_g}{\partial y}\frac{\partial \theta}{\partial y} \end{pmatrix}$$

Advantages:

- Only *one forcing term* → no partial cancellation of opposite forcing terms
- All forcing can be evaluated on a *single isobaric surface* → should use multiple levels
- Can be easily computed from 3-D data fields (quantitative)
- The Q-vectors *computed from numerical model output* can be plotted on maps to obtain a clear representation of synoptic-scale vertical motion

Disadvantages:

- Can be *very difficult to estimate from standard upper-air observations*
- Neglects diabatic heating
- Neglects orographic effects

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